Study of thermo-elastic effects in laminated composites by the cohesive zone model

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Introduction

- Cohesive Zone Model

- Extension to coupled thermoelastic problems

Traction-opening relations are taken into account and combined with heat flux-opening relations based on contact mechanics

The purpose is twofold :

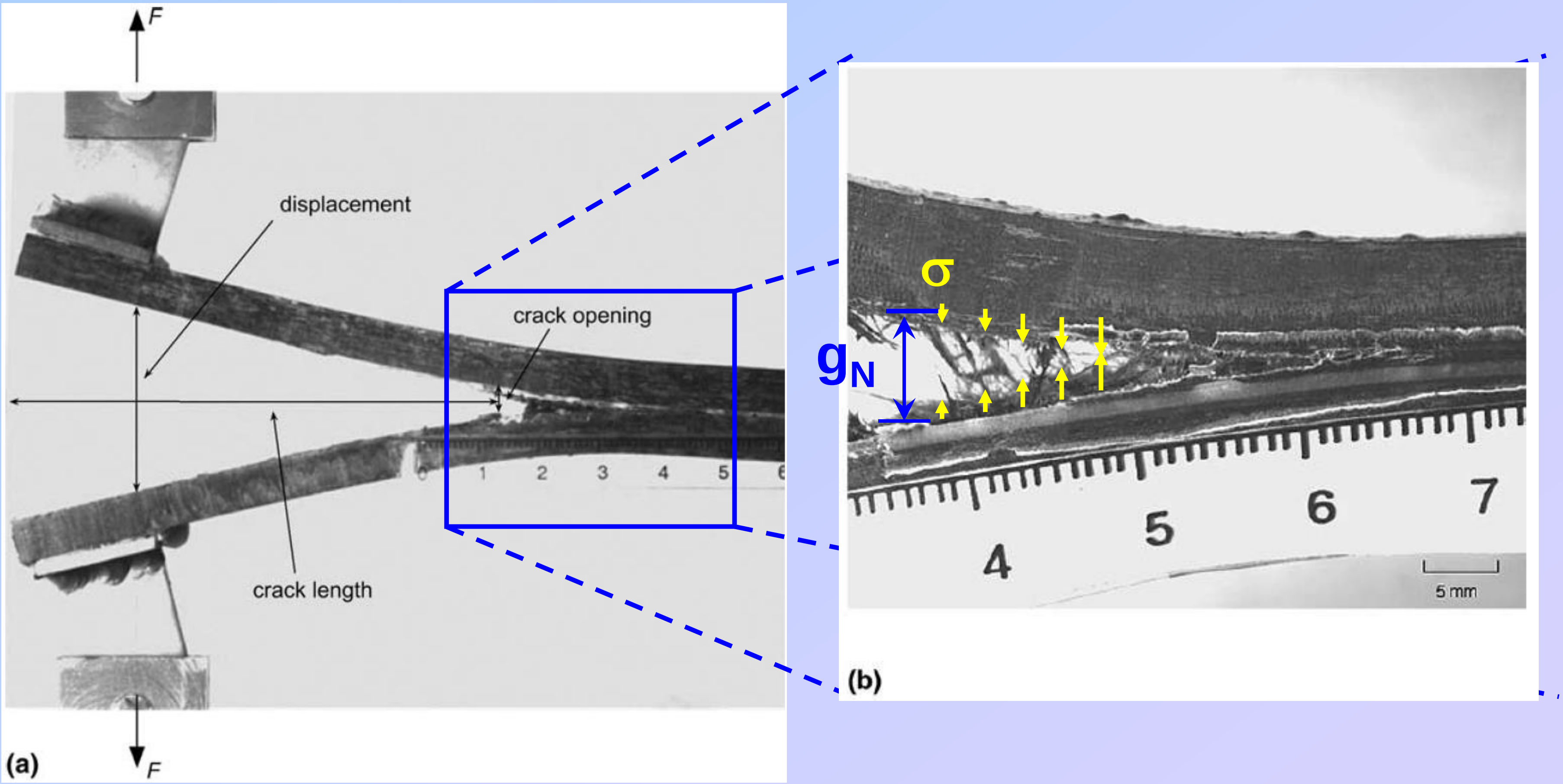
1. To generalize thermoelastic contact analysis to fracture
(Zavarise et al. 1992; Wriggers and Zavarise, 1993)

2. To propose a thermal model more physically sound than the others presented in the Literature (es. Kapitza's model)
(Hattiangadi&Siegmund 2004; Yvonnet et al. 2010; Özdemir et al., 2010)

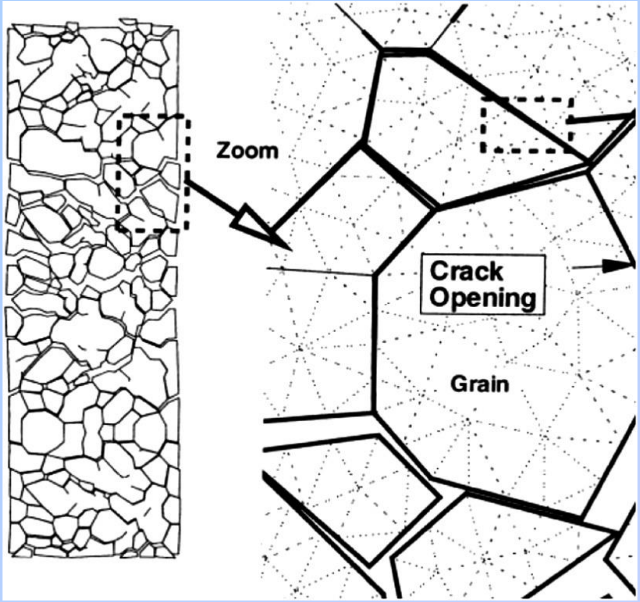
- Conclusions

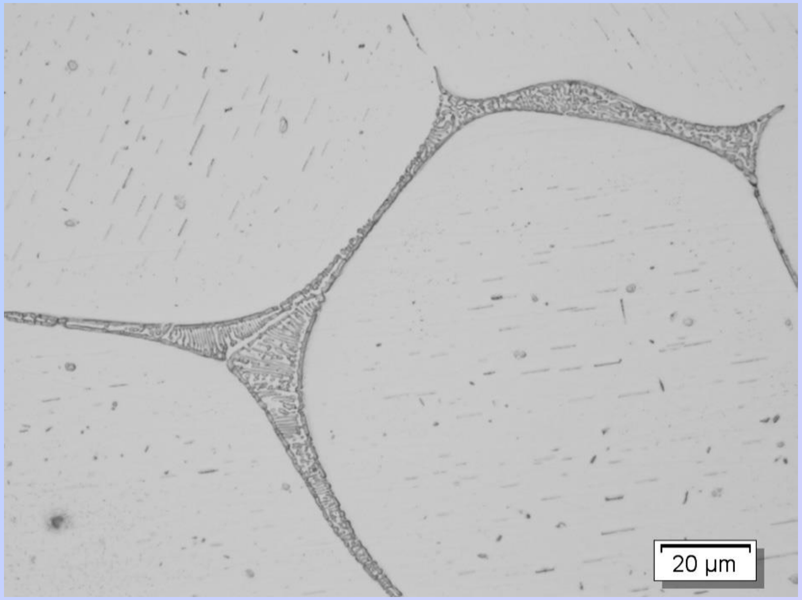
Cohesive Zone Model (CZM)

Double cantilever beam test



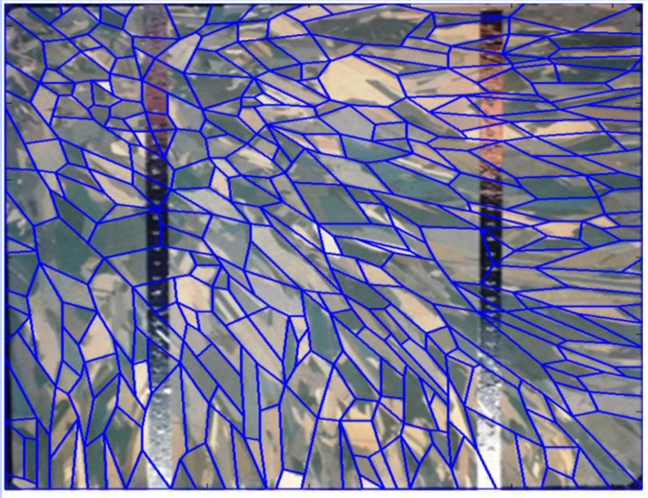
CZM: applications





MgCa0.8: biomedical stents

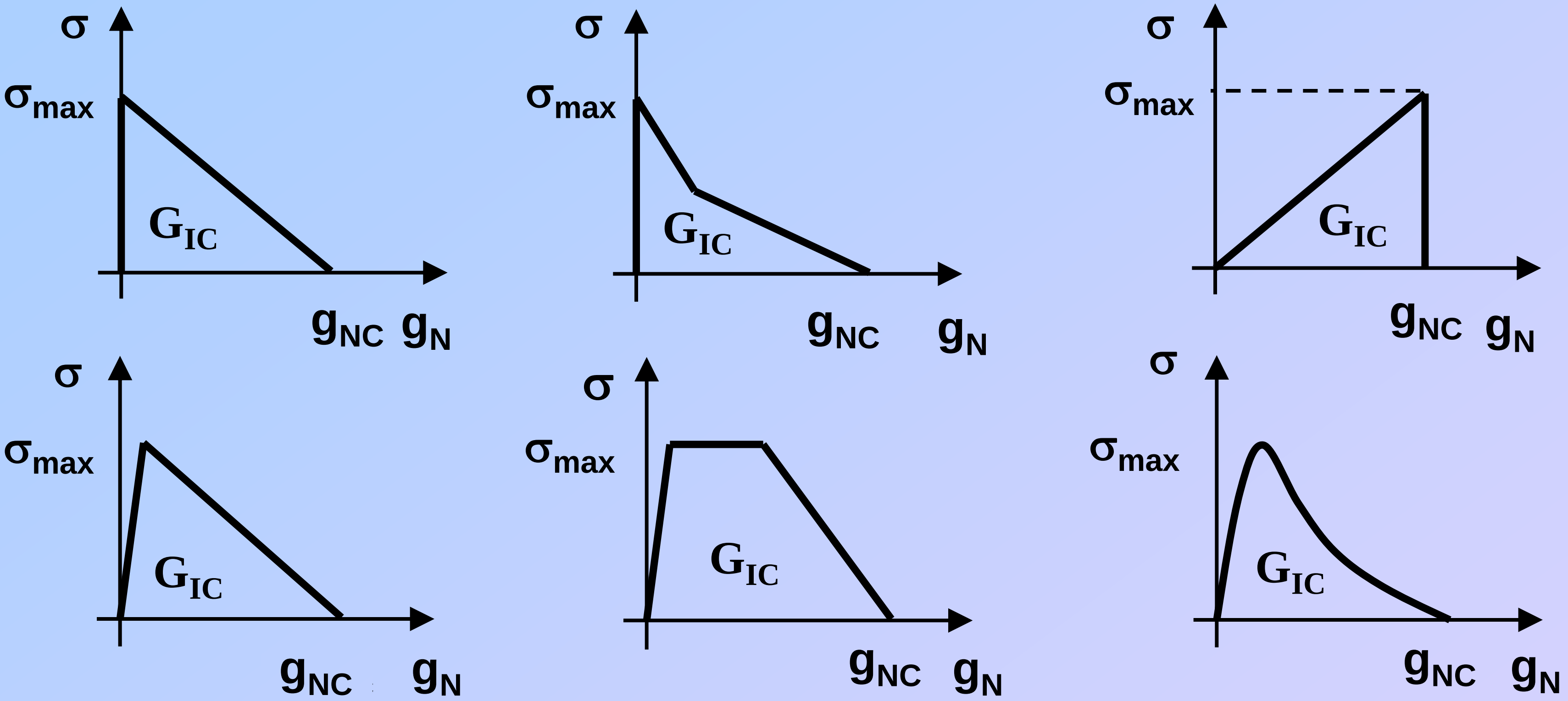
Paggi et al., *Computational Mechanics* (2012)



Silicon cells: photovoltaic modules

Paggi and Sapora, *Energy Procedia* (2013)

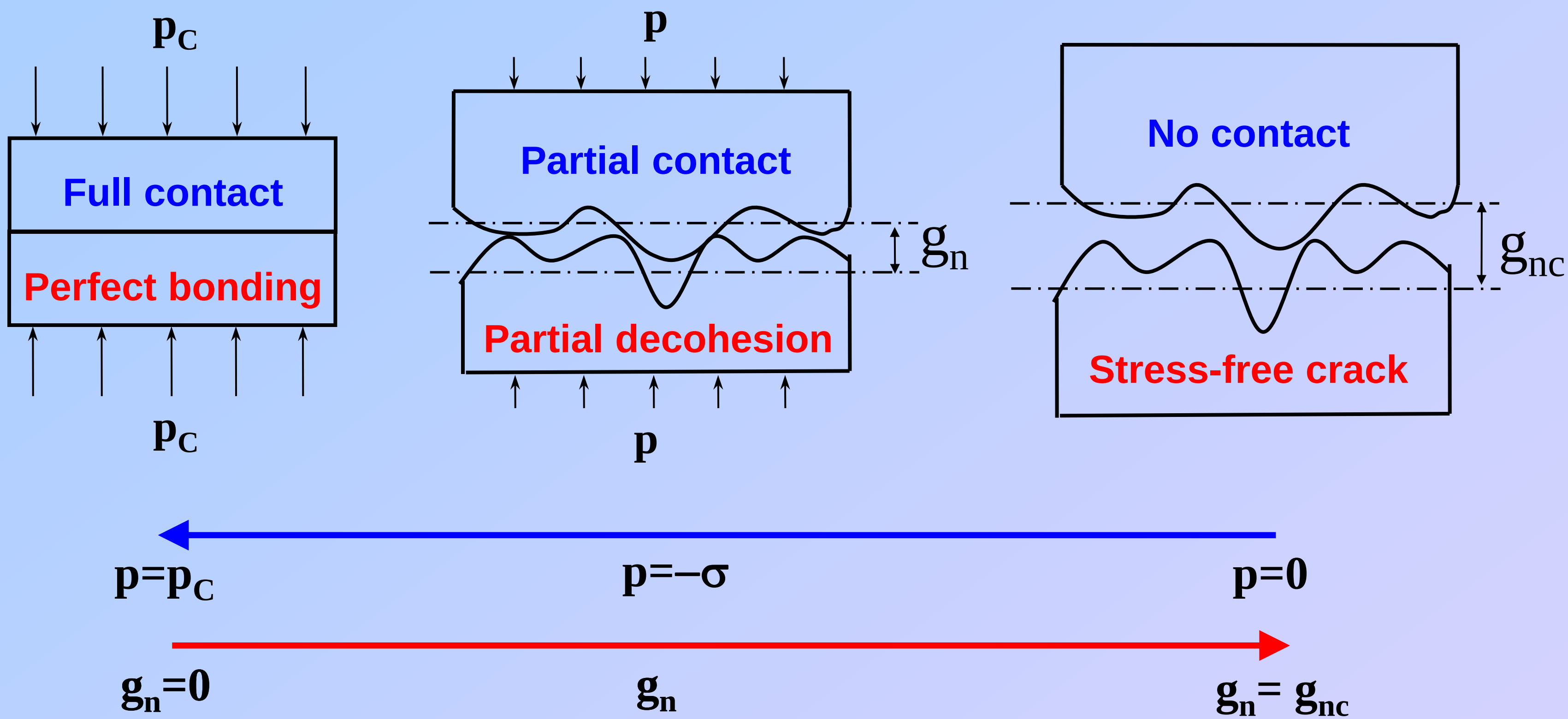
CZM: constituive law



Open issues:

How is possible to relate the shape of the CZM to physics/mechanics?

Fracture / contact mechanics

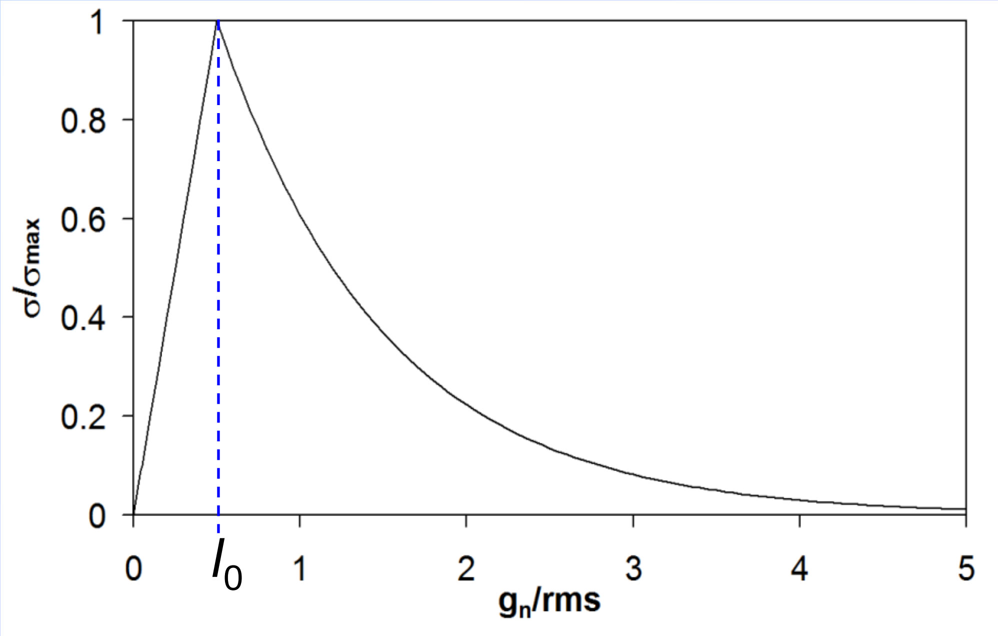


The introduction of “rms”, the root mean square of asperity heights reveals to be necessary for an accurate analysis..

Paggi and Barber, *International Journal of Heat and Mass Transfer* (2011)

Cohesive stress

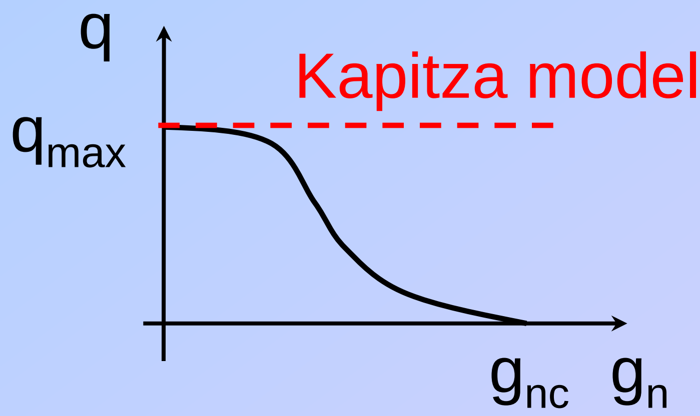
$$\sigma = \begin{cases} \sigma_{max} \exp\left(\dfrac{-l_0 - g_t}{\mathrm{rms}}\right) \dfrac{\mathrm{g_n}}{l_0} & \text{if} \quad 0 \le \dfrac{g_n}{\mathrm{rms}} < \dfrac{l_0}{\mathrm{rms}} \\ \sigma_{max} \exp\left(\dfrac{-g_n - g_t}{\mathrm{rms}}\right) & \text{if} \quad \dfrac{l_0}{\mathrm{rms}} \le \dfrac{g_n}{\mathrm{rms}} < \dfrac{g_{nc}}{\mathrm{rms}} \\ 0 & \text{if} \quad \dfrac{g_n}{\mathrm{rms}} \ge \dfrac{g_{nc}}{\mathrm{rms}} \end{cases}$$



Greenwood and Williamson, *Proceedings of the Royal Society of London* (1966)

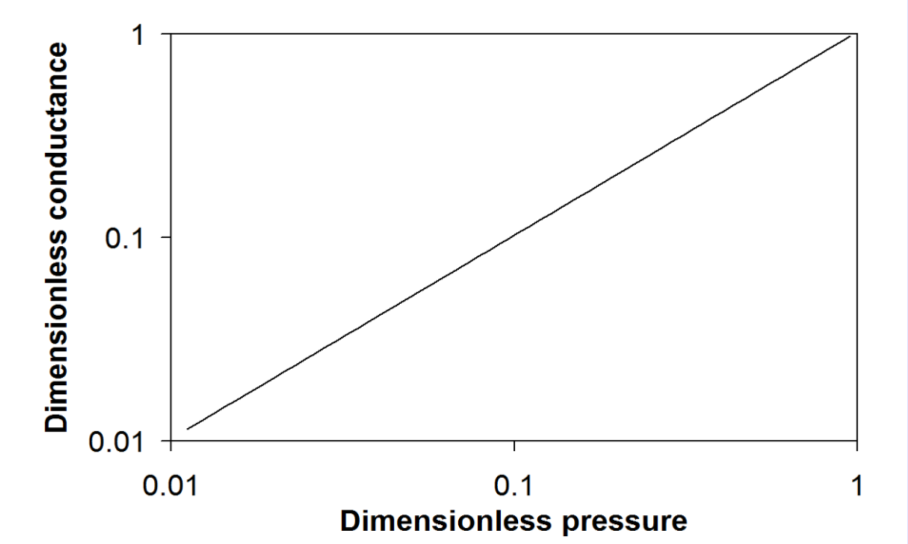
Cohesive heat flux

$$q = -k_{int}(g_n)\,\Delta T$$



N.B. Interface conductance proportional to the normal stiffness:

$$k_{\text{int}} = \begin{cases} \dfrac{1}{r_{\text{int}}} & \text{if} \quad 0 \le \dfrac{g_n}{\text{rms}} < \dfrac{l_0}{\text{rms}} \\[2ex] \dfrac{2\sigma}{r_{\text{int}} E_{\text{int}}\,\text{rms}} & \text{if} \quad \dfrac{l_0}{\text{rms}} \le \dfrac{g_n}{\text{rms}} < \dfrac{g_{nc}}{\text{rms}} \\[2ex] 0 & \text{if} \quad \dfrac{g_n}{\text{rms}} \ge \dfrac{g_{nc}}{\text{rms}} \end{cases}$$

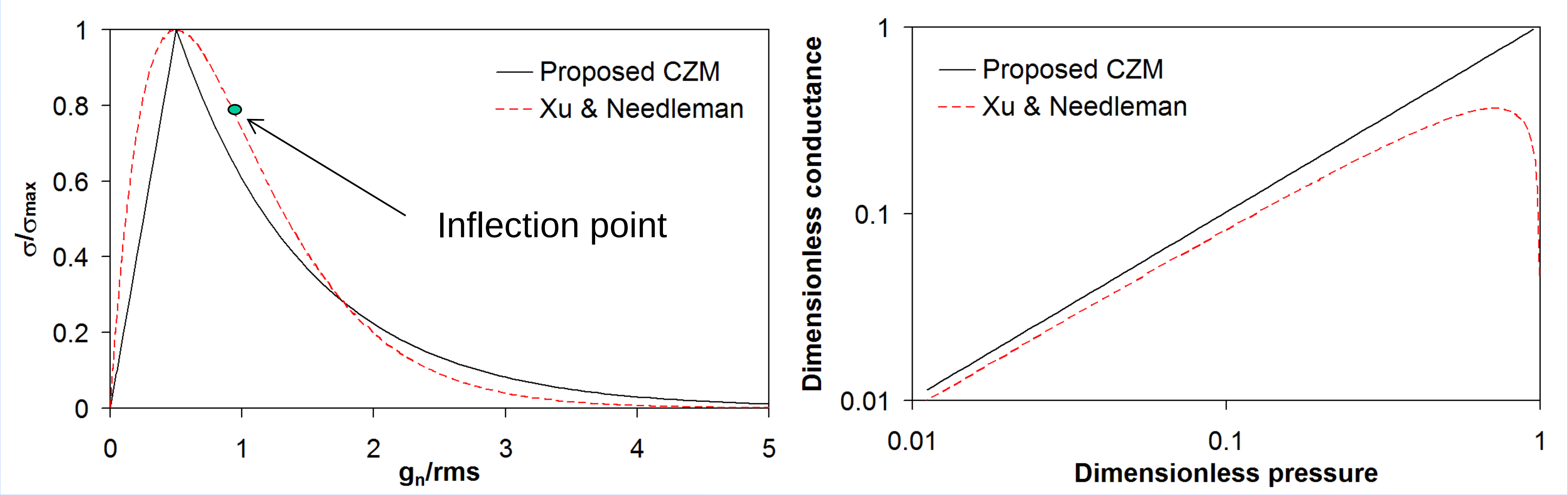


Barber, *Proceedings of the Royal Society of London* (2003)

Comparison with other models

Stress

Conductance

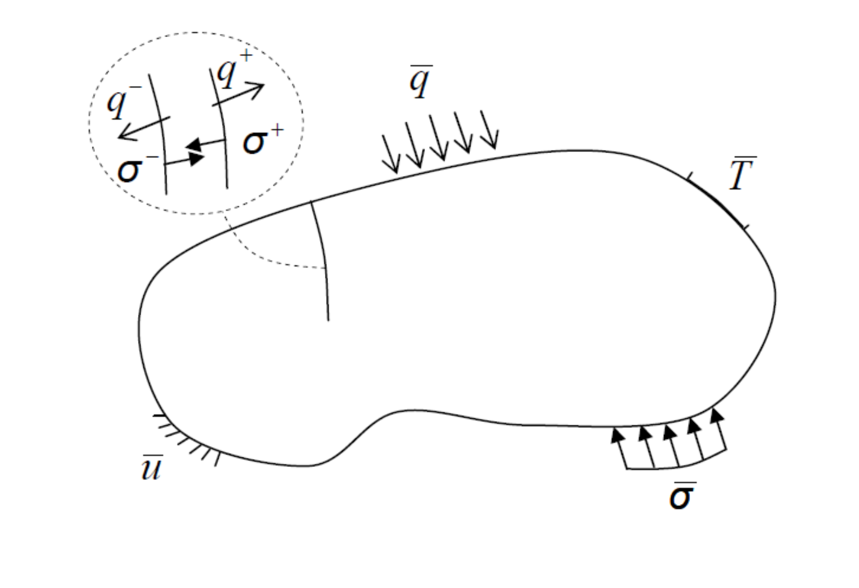


($l_0/rms = 0.5$)

Xu and Needleman, *Journal of the Mechanics and Physics of Solids* (1994)

Özdemir et al. *Computational Mechanics* (2010)

Formulation of the problem: 1



V: volume

S: surface

S: Cauchy stress tensor

f: body force vector

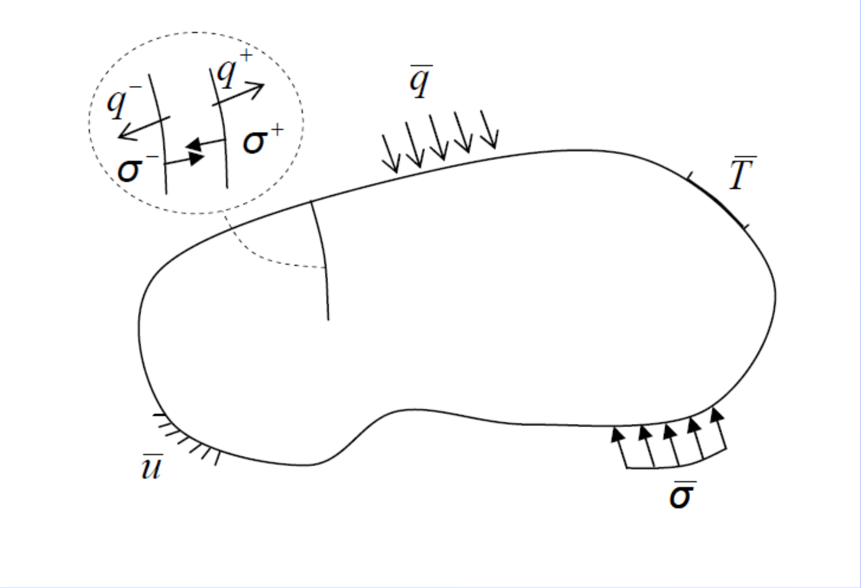
w: displacement vector

Strong form: $\nabla^T \mathbf{S} + \mathbf{f} = \mathbf{0}$

Weak form (PVW):

$$\int_V \mathbf{S} : \nabla(\delta \mathbf{w}) \mathrm{d}V = \int_V \mathbf{f}^T (\delta \mathbf{w}) \mathrm{d}V + \int_S \bar{\boldsymbol{\sigma}}^T (\delta \mathbf{w}) \mathrm{dS} + \int_{S_{\mathrm{int}}} \boldsymbol{\sigma}^T (\delta \mathbf{w}) \mathrm{dS}$$

Formulation of the problem: 2



V: volume

S: surface

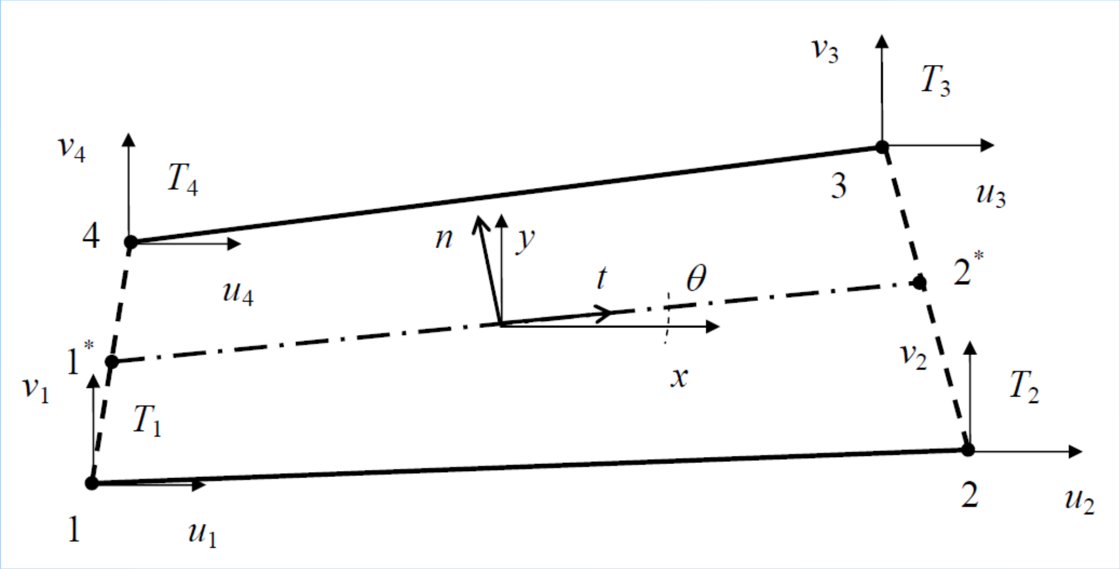
q: heat flux vector
Q: heat generation
T: temperature

Strong form: $$-\nabla^T \mathbf{q} + Q = \rho c \dot{T}$$

Weak form (variational form of the energy balance):

$$\int_V \mathbf{q}^T \nabla(\delta T)\mathrm{d}V = \int_V (\rho c \dot{T} - Q)\delta T \, \mathrm{d}V + \int_S \bar{\mathbf{q}}^T (\delta \mathbf{w}) \, \mathrm{dS} + \int_{S_{\mathrm{int}}} q(\delta T) \, \mathrm{dS}$$

FEA: interface element



Gap vector:
$\boldsymbol{g}=(g_t,g_n,g_T)^T$

Flux vector:
$\boldsymbol{p}=(\tau,\sigma,q)^{\mathrm{T}}$

Weak form for the interface elements:

$$\delta G_{\text{int}} = \int_{S_{\text{int}}} \delta\mathbf{g}^T \mathbf{p}\, \mathrm{d}S$$

Consistent linearization of the interface constitutive law (quadratic convergence in the Newton-Raphson scheme):

$$\mathbf{p} = \mathbf{C}\,\mathbf{g} \quad \Longrightarrow \quad \delta G_{\text{int}} = \int_{S_{\text{int}}} \delta\mathbf{g}^T \mathbf{C}\,\mathbf{g}\, \mathrm{d}S$$

Paggi M, Wriggers P. *Comp. Mat. Sci.* (2012)

FEA: constitutive matrix C

$$\mathbf{C} = \begin{bmatrix} \dfrac{\partial \tau}{\partial g_t} & \dfrac{\partial \tau}{\partial g_n} & 0 \\ \dfrac{\partial \sigma}{\partial g_t} & \dfrac{\partial \sigma}{\partial g_n} & 0 \\ \dfrac{\partial q}{\partial g_t} & \dfrac{\partial q}{\partial g_n} & \dfrac{\partial q}{\partial g_T} \end{bmatrix}$$

CZM

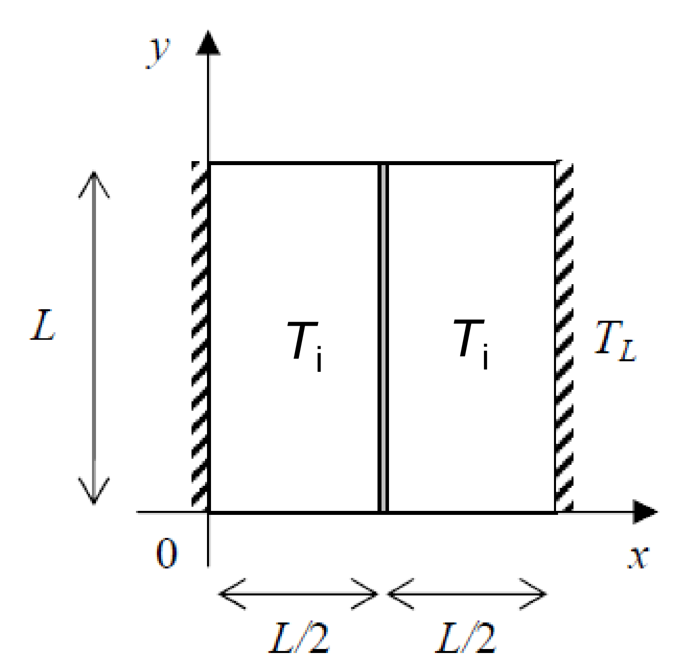
Thermoelastic effect

N.B. if we consider k_{int}*=const* (Kapitza's model): $\dfrac{\partial q}{\partial g_t} = \dfrac{\partial q}{\partial g_n} = 0$

Tangent matrix for the Newton Raphson algorithm:

$$\mathrm{K} = \mathrm{R}^T \int_{S_{int}} \mathrm{B}^T \mathrm{C} \mathrm{B} \, dS \mathrm{R}$$

Parametric analysis



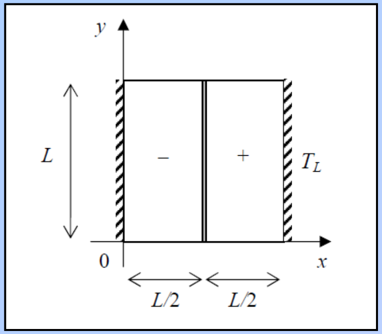
$T_L < T_i$
$y^* = y/L = 0.5$
$I_0/rms = 0.01$
$\nu = 0.1$

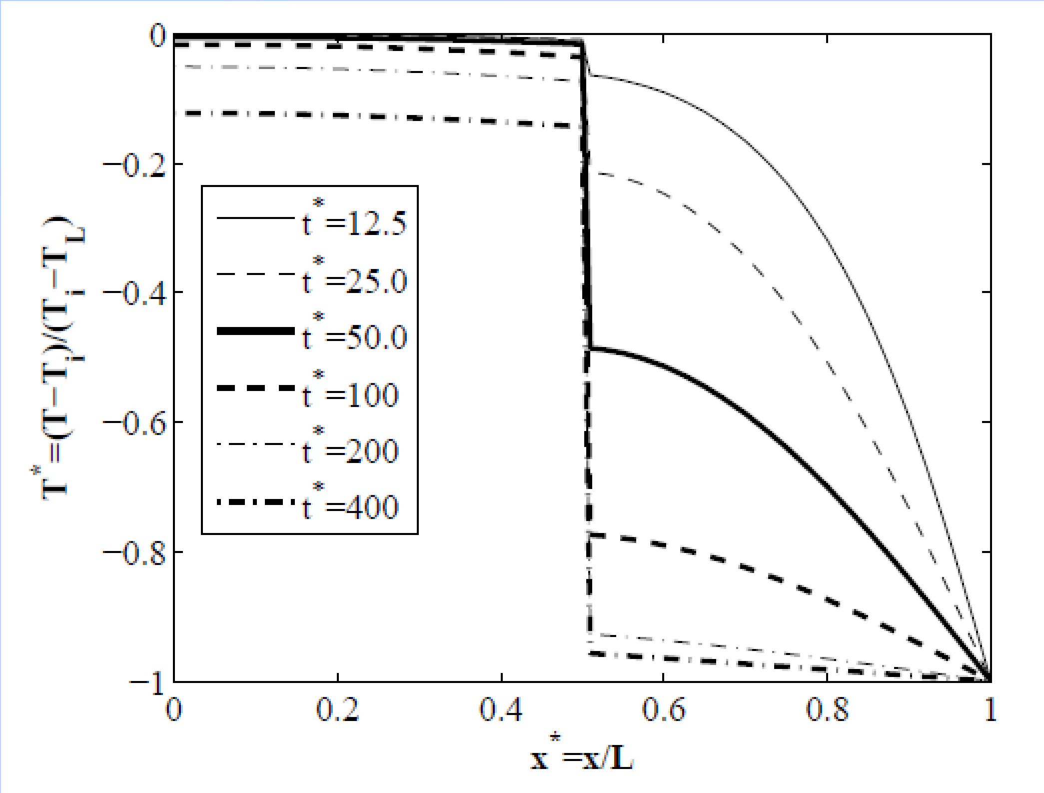
Let us focus our attention on the transient regime. Buckingham's Theorem allows us reducing the dependency of T^* (combination of T, T_i and T_L) to four parameters:

$$T^* = T^*\left(x^* = \frac{x}{L}, t^* = \frac{tD}{L^2}, g_{nc}^* = \frac{g_{nc}}{L}, \sigma_{max}^* = \frac{\sigma_{max}}{E}\right)$$

Results 1

Temperature field as time varies ($y^* = y/L = 0.5$)

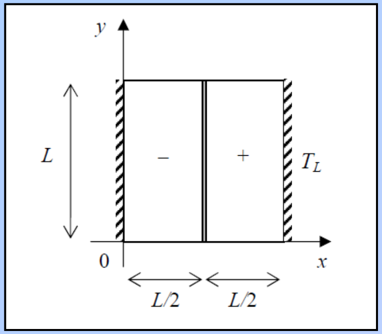


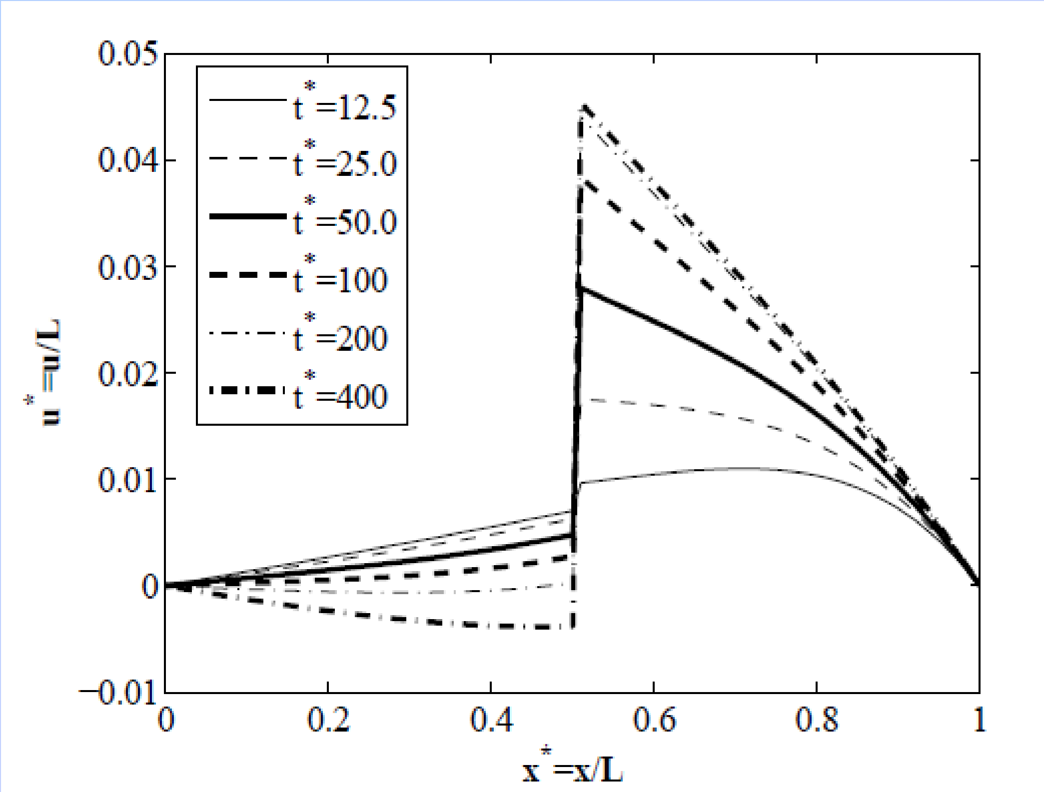


$\sigma^*_{max} = 0.032$, $g^*_{nc}=0.05$

Results 2

Displacement field as time varies ($y^* = y/L = 0.5$)





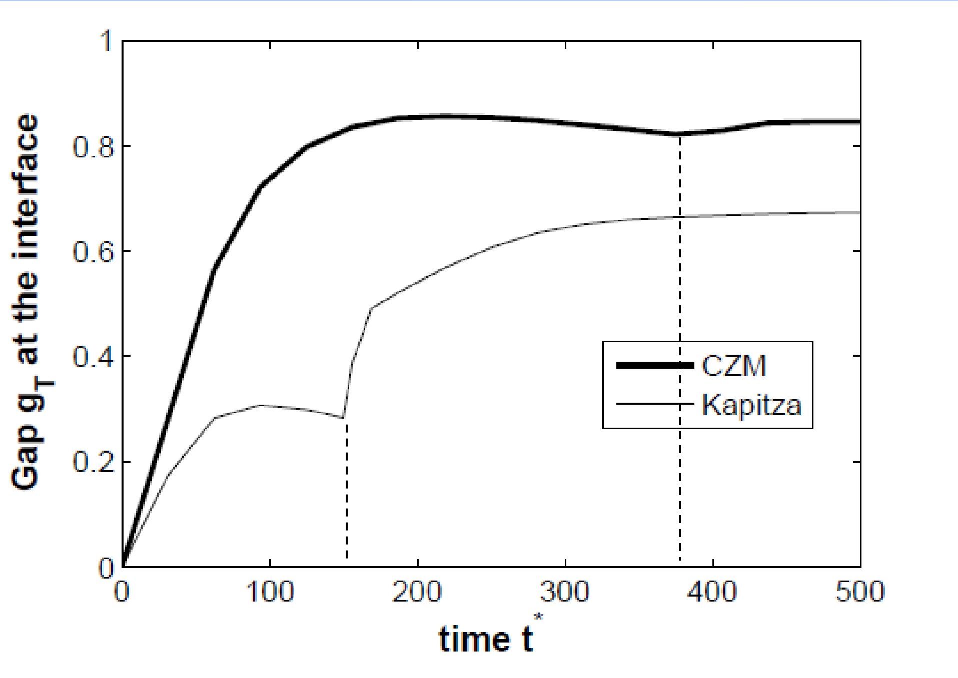
$\sigma^*_{max} = 0.032, \;\; g^*_{nc} = 0.05$

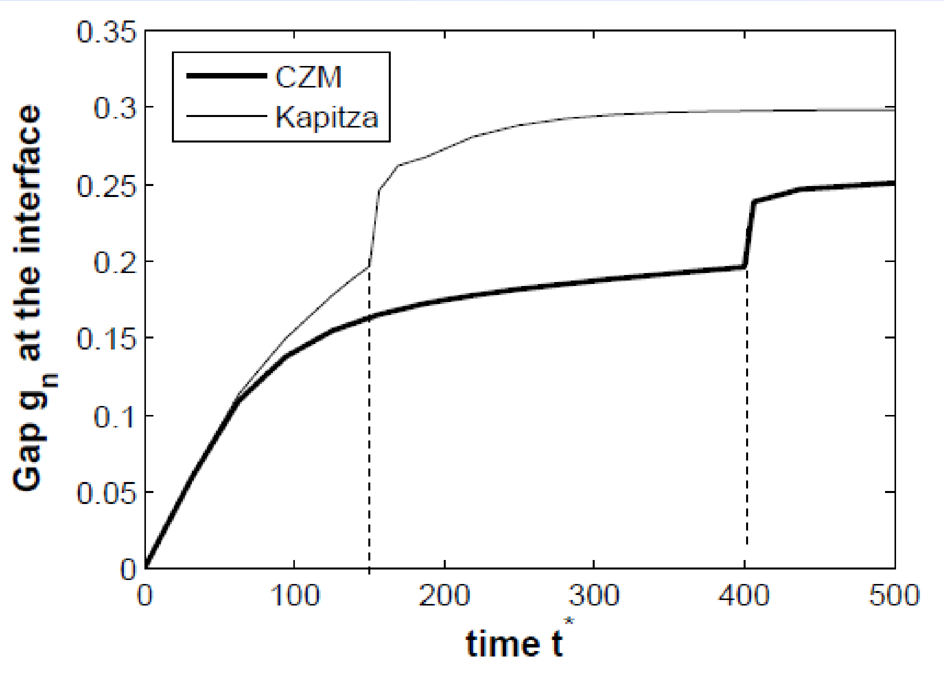
Results 3

Let us compare the CZM results with $k_{int}=1/r_{int}$:

TEMPERATURE GAP

DISPLACEMENT GAP

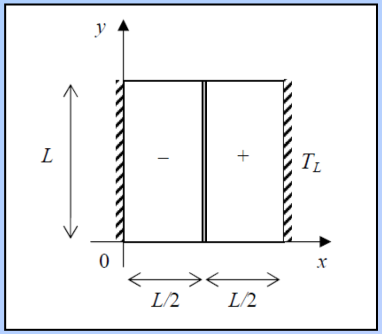


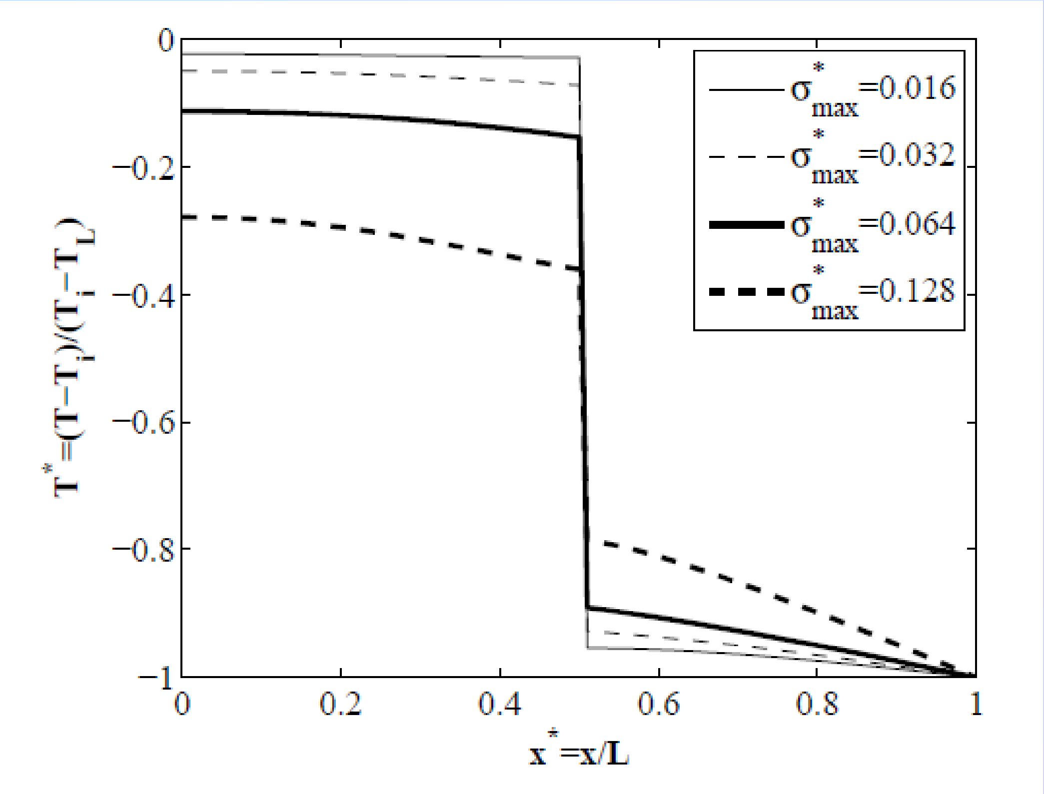


Neglecting thermoelastic coupling leads to different thermal and elastic behaviors..

Results 4

Temperature field as σ^*_{max} varies (t^*=200)





An analogous behavior is observed by modifying g^*_{nc} (σ^*_{max} *fixed*)

Conclusions

A thermo-mechanical CZM inspired by contact mechanics between rough surfaces has been put forward: the interface conductivity results to be a function of cohesive stresses

Thermo-elastic effects related to the transient regime are investigated, with particular attention to:
i) the time evolution of the temperature and displacement
ii) the influence of the cohesive parameters on fracture initiation

Future perspectives:

i) numerical solution of 2D problems in the presence of multiple micro-cracks
ii) coupling between the electric and the thermal fields

Acknowledgements

FIRB Future in Research

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for Renewable Energy Applications, RBFR107AKG

Thanks
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