institute IMDEA materials

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Fracture in discrete systems simulated by MD-FEM: the role of nonlocality and its effect on flaw-tolerance

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IMT INSTITUTE FOR ADVANCED STUDIES LUCCA

Outline

- **Motivations**
  - Analyze an equivalent discrete system with long range interactions to detect nonlocal behaviour at the continuum level
- **Computational Model**
  - Nonlocal effect in discrete systems
  - Nonbonded interaction generalized potential
  - FE model based on large displacement formulation
- **Numerical examples**
  - Monodimensional Case study
  - Bidimensional Case Study
    - ✓ Flaw-tolerance
    - ✓ Force-distribution
- **Conclusions**
- **Coming progresses**

Scenario

Nonlocal *continuum* theories to describe long-range interactions

Gradient models

Integral formulations

Fractional calculus

•M. Di Paola, M. Zingales, Int J Solids Struct, 45: 5642-5659, 2008

•A. Carpinteri, P. Cornetti, A. Sapora, M. Di Paola, M. Zingales, Phys Scripta, T136: 014003-014010, 2009

•Z. Bazant, J. Eng. Mech., 120(3), 593–617

•A.C. Eringen, Springer Verlag, 2002

Discrete systems of particles or molecules to describe long-range interactions

Lattice beam models

Born models

Spherical elements models

Molecular Dynamics

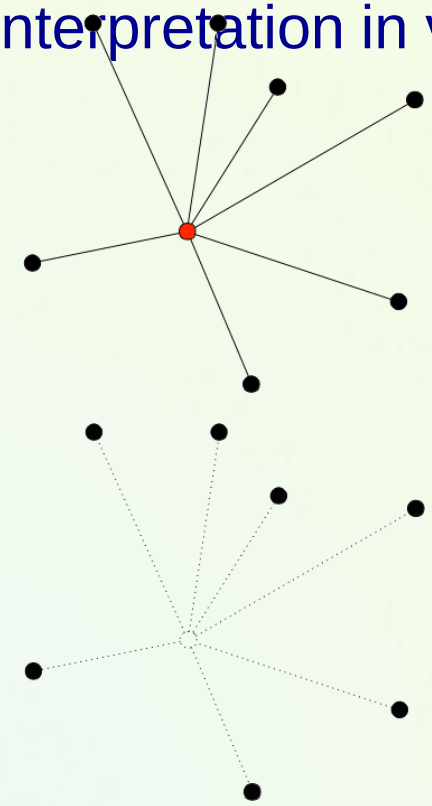
- V.E. Tarasov, Journal of Physics A 39: 14895-14910, 2006
- J.G.M. van Mier, E. Schlangen, A. Vervuurt, Continuum Models for Materials with Microstructure: 341-377, 1995
- A. Parisi, G. Caldarelli, L. Pietronero, EPL, 52 (3): 304-310, 2000
- J.G.M. van Mier, Int J Fracture, 143: 41-78, 2007

New model proposal

Objectives:

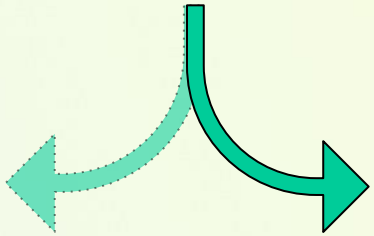
1)Nonlocal behaviour prediction for continua by upscaling the mechanical behaviour of discrete nonlocal structures

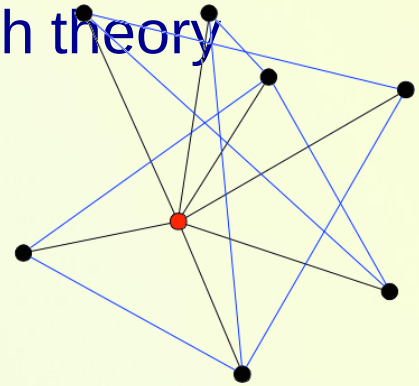
2)Interpretation in view of network and graph theory



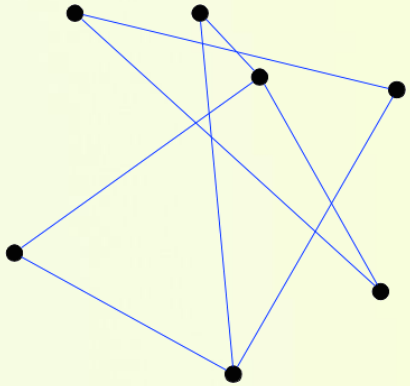
Node Removal

Nonlocal effects



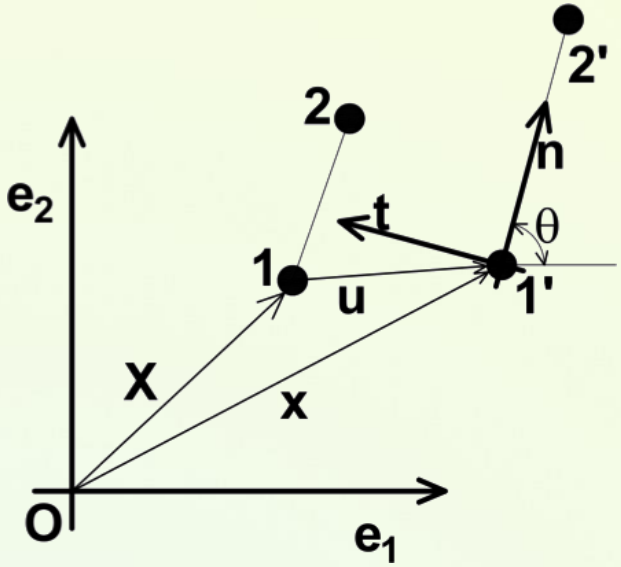


Redundancy



Numerical method

Discrete system of atoms or molecules



$$\mathbf{X} = (X_1, Y_1, X_2, Y_2)^T$$ Initial coord. in global system

$$\mathbf{u} = (u_1, v_1, u_2, v_2)^T$$ Element displacement vector

$$\mathbf{x} = \mathbf{X} + \mathbf{u}$$ Coord. in updated config. (large displ.)

$$\mathbf{g}_{loc} = \mathbf{RLx}$$ Gap vector in local ref. system

R= rotation matrix

L=operator matrix

Force vector in local system $\mathbf{F}_{loc} = (F_{LJ}, 0)^T$

P. of V.W. linearization to apply Newton-Raphson procedure

Residual vector $$\mathbf{p} = \left(\mathbf{RL} + \frac{\partial \mathbf{R}}{\partial \mathbf{u}} \mathbf{Lu} \right)^T \mathbf{F}_{loc}$$

Third order tensor

Stiffness matrix $$\mathbf{K} = \mathbf{K}_{mat} + \mathbf{K}_{geom}$$

FEAP

Finite Elements Analysis Program

M. Paggi, P. Wriggers (2011) Comp. Mat. Sci., 50:1634-1643

Constitutive law: L-J potential

$$E_{tot} = \cancel{E_b} + E_{nb}$$

$$P_{LJ} = 4e\left[\left(\frac{r_0}{g_n}\right)^a - \left(\frac{r_0}{g_n}\right)^b\right]$$

e = depth of potential well

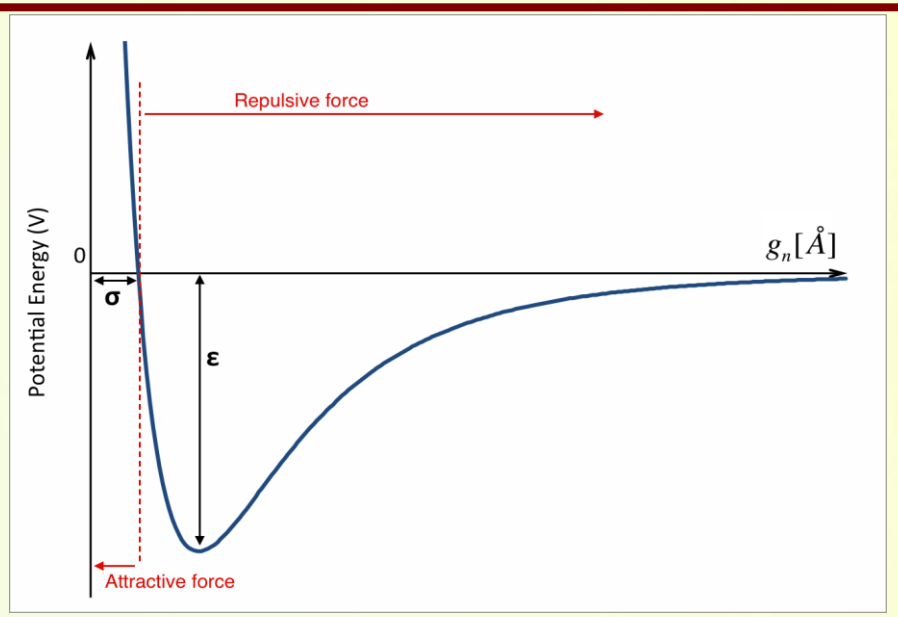
r_0 = equilibrium distance

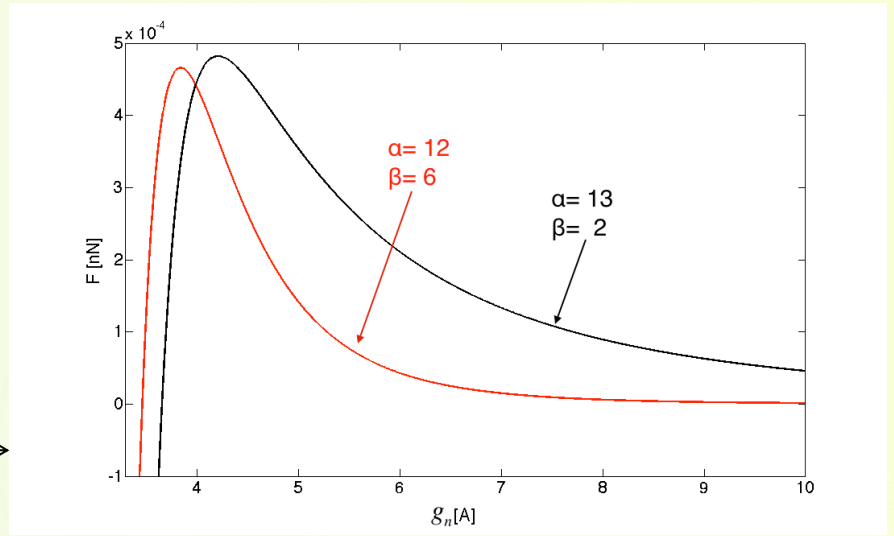
$$F_{LJ}(g_n) = -\nabla P_{LJ}(g_n) = -\frac{dP_{LJ}(g_n)}{d(g_n)}$$

$$= -4e\left[a\left(\frac{s^a}{g_n^{a+1}}\right) - b\left(\frac{s^b}{g_n^{b+1}}\right)\right]$$

Generalized formulation

a = 13 b = 2





L.Y. Jiang et al. (2006) J. Mech. Phys. Solids 54: 2436-2452

Monodimensional Model

NLI = NonLocality Index

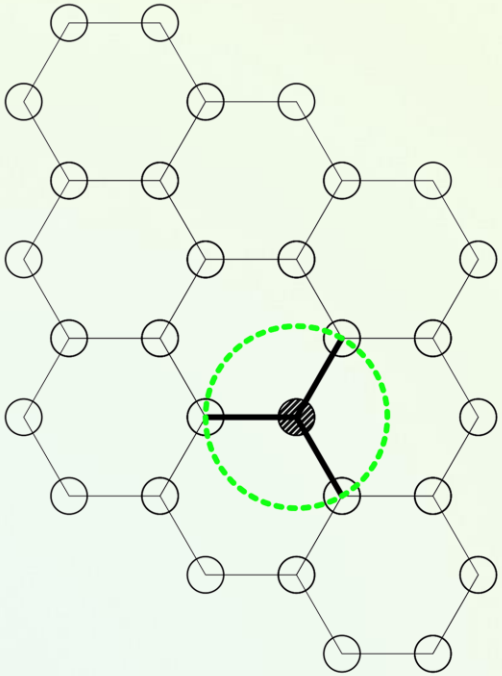
$$NLI = \frac{max(l_i)}{l_0}$$

l_i = length of the i-th link

l_0 = local link length

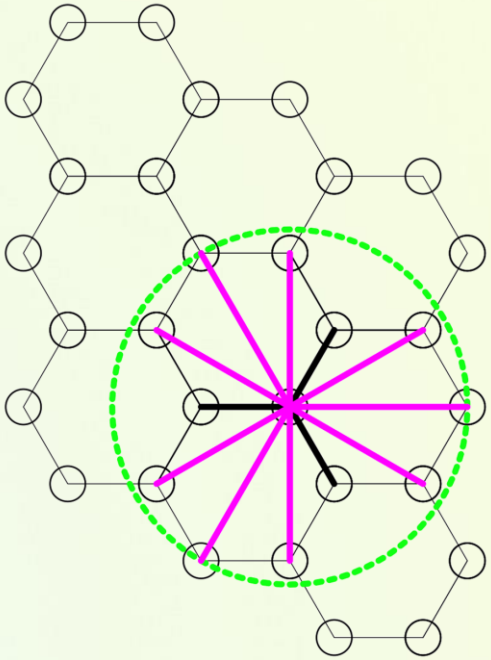
Local system

NLI = 1

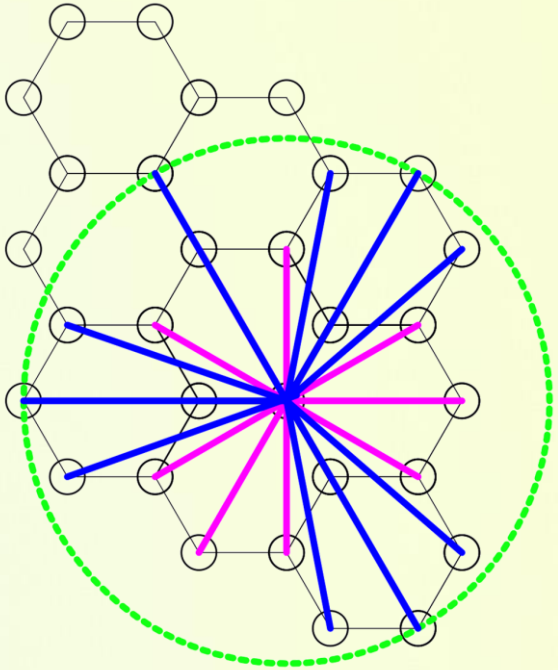


Nonlocal systems

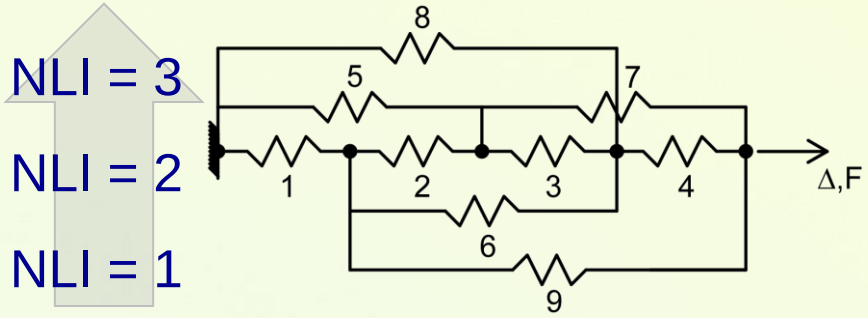
NLI = 2



NLI =3

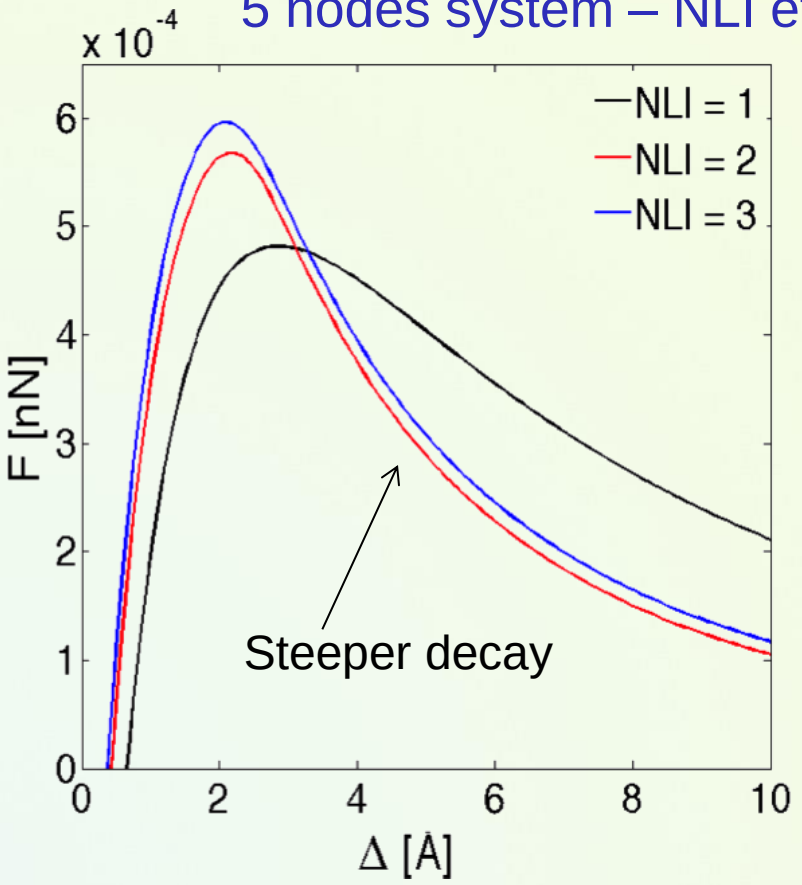


Monodimensional Model

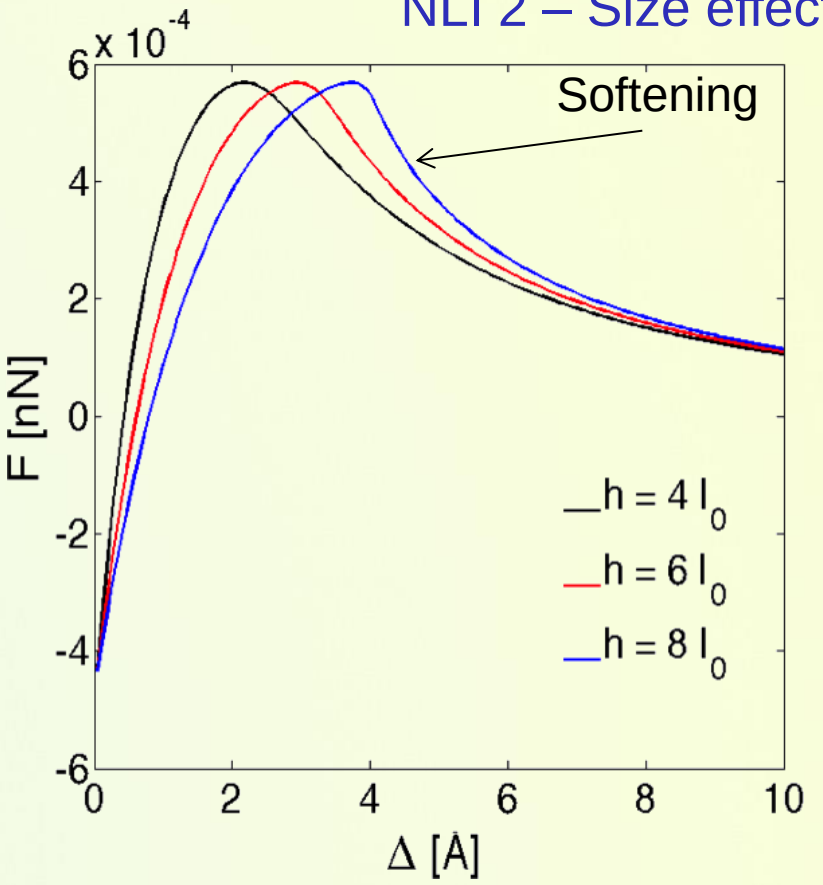


Global force vs. global displacement

5 nodes system – NLI effect



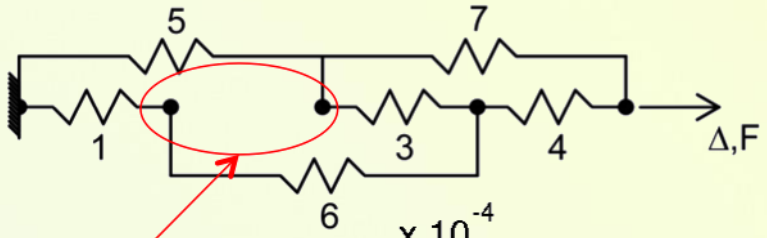
NLI 2 – Size effect



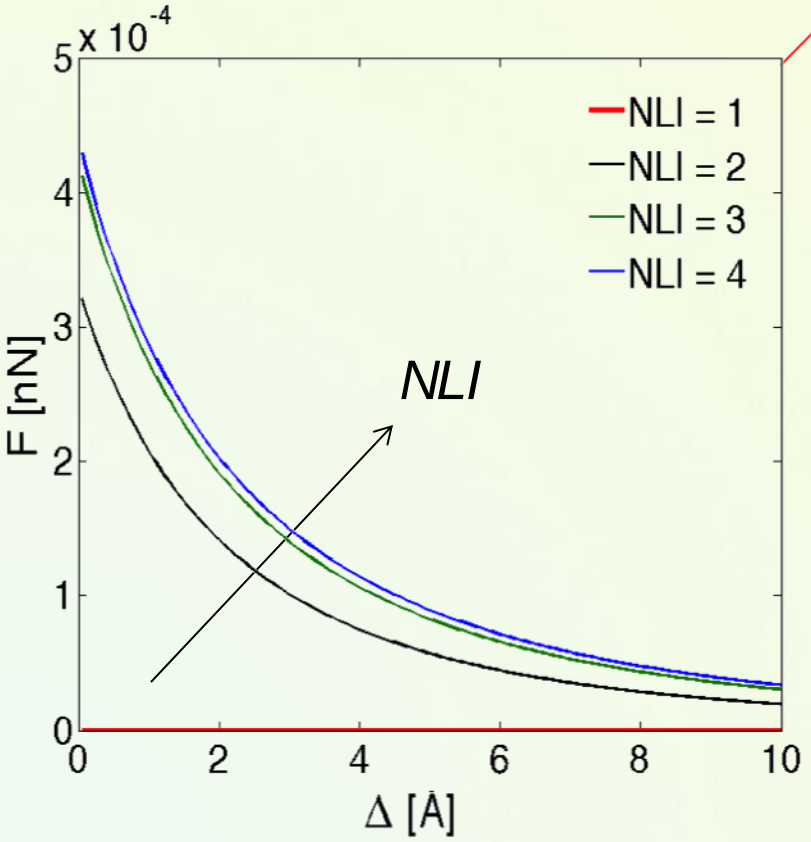
Monodimensional Model - with defects

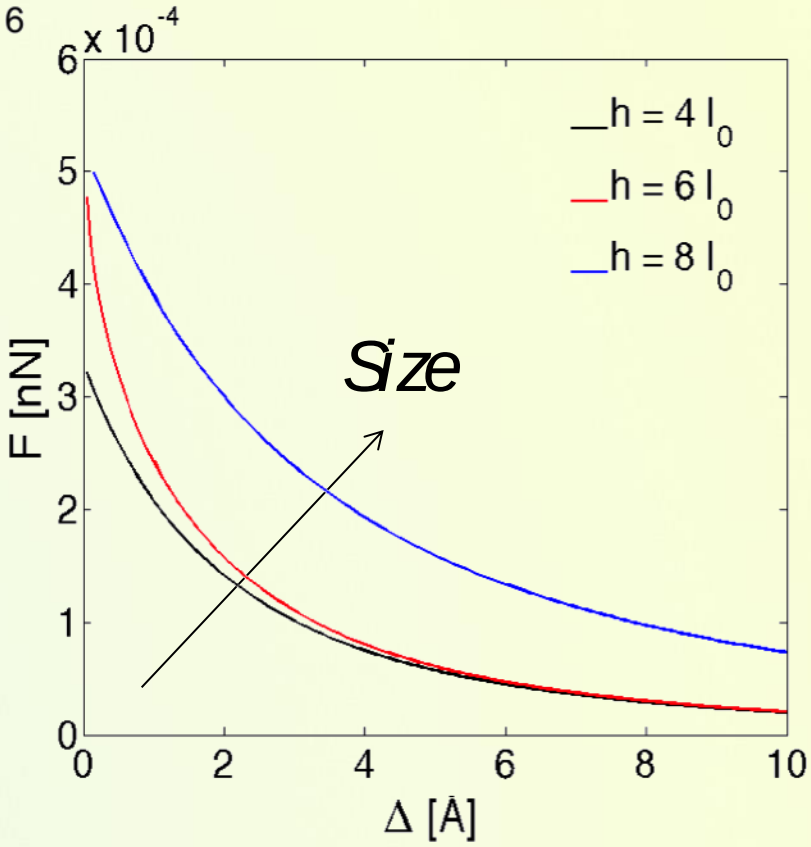
Global force vs. global displacement

5 nodes system
NLI effect



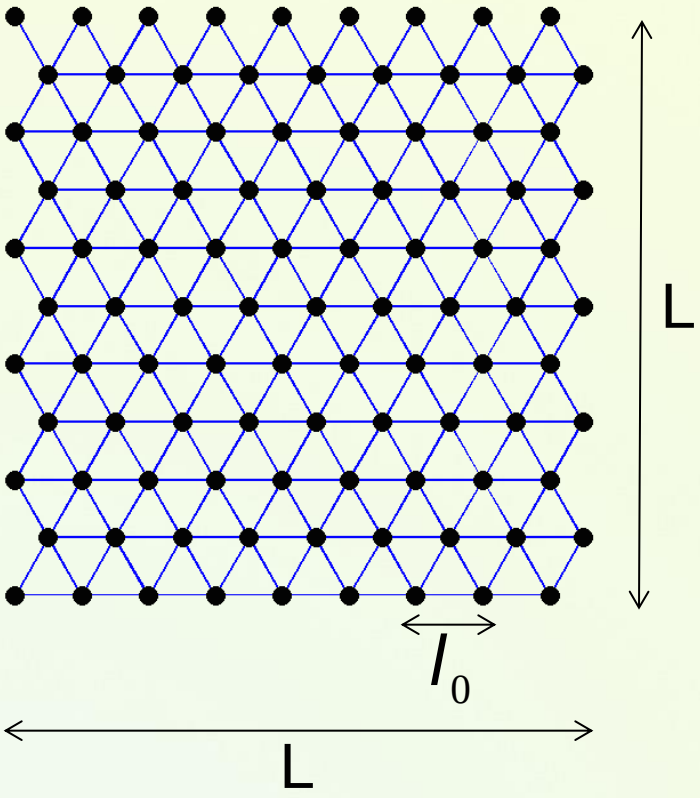
NLI=2
Size effect





Defect better tolerated with increasing NLI and system size

Bidimensional Model

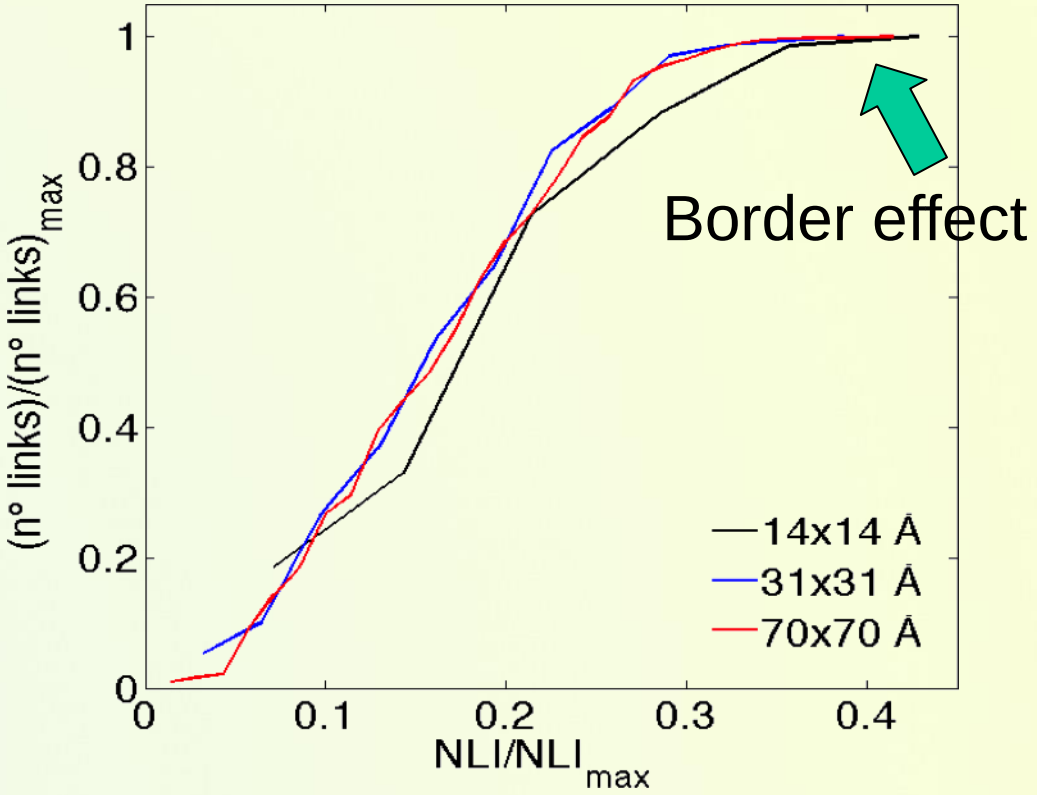


Hexagonal mesh

$l_0 = 3.5\text{Å}$ $\quad NLI = 1 \longrightarrow NLI_{max}$

L [Å]	NLI_{max}	n. links
14	6	600
31	12	4851
70	29	126756

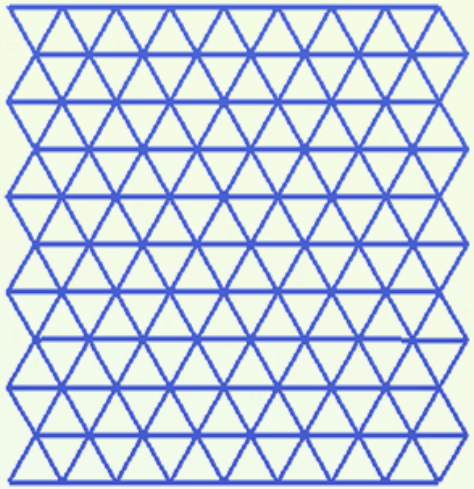
Relation between NLI and n° links

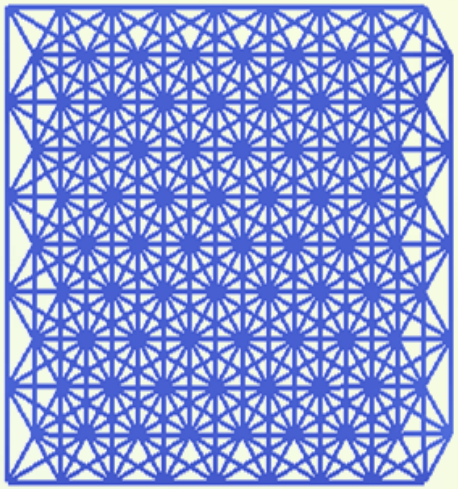


Bidimensional Model

ParaView
Parallel Visualization Application

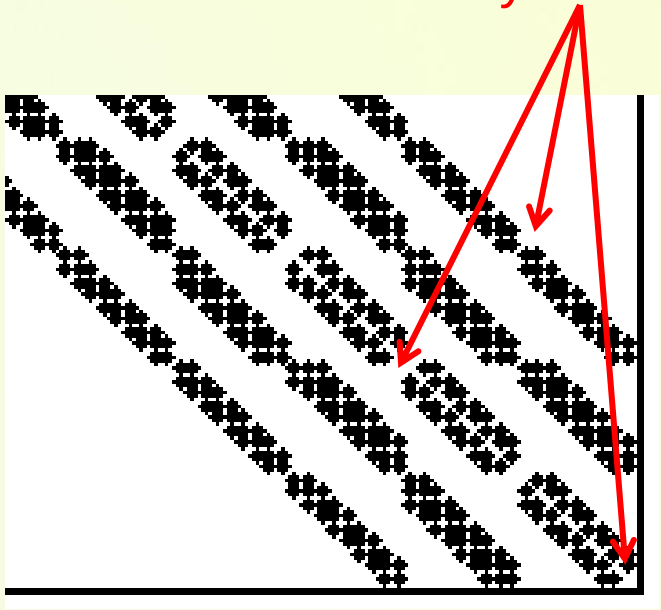
Connectivity Matrix: links increase with NLI

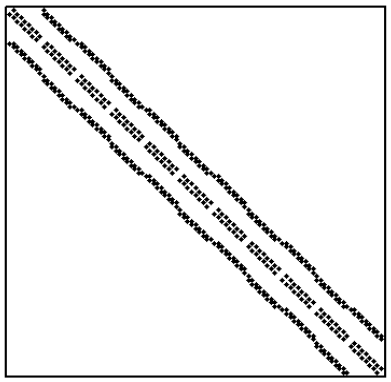


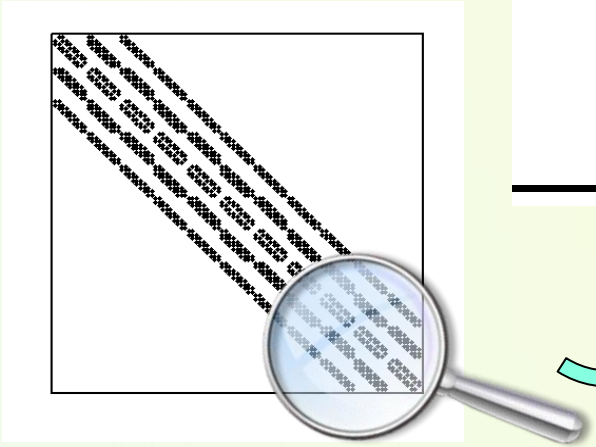


NLI=1 ⟶ NLI=3

Border effects due to finite size of the system





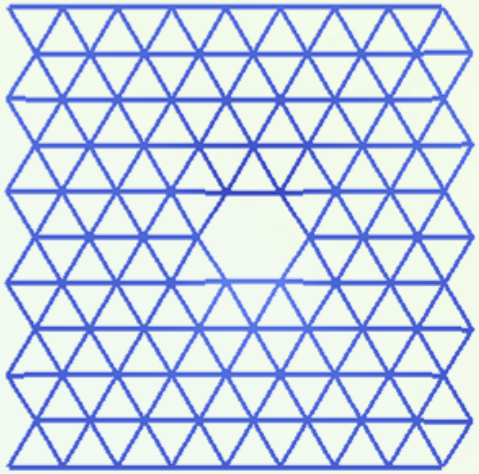


Bidimensional Model - with defects

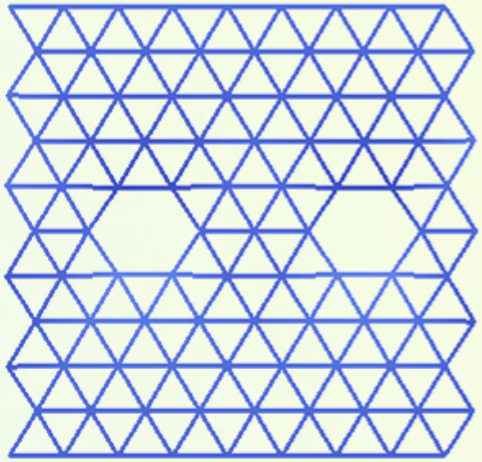
Flaw-tolerance study

Evaluation of the influence of NLI and defect position

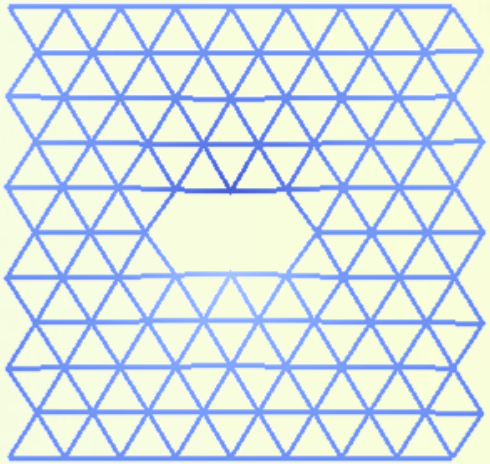
Case 1
One node removal



Case 2
Two non adjacent nodes removal

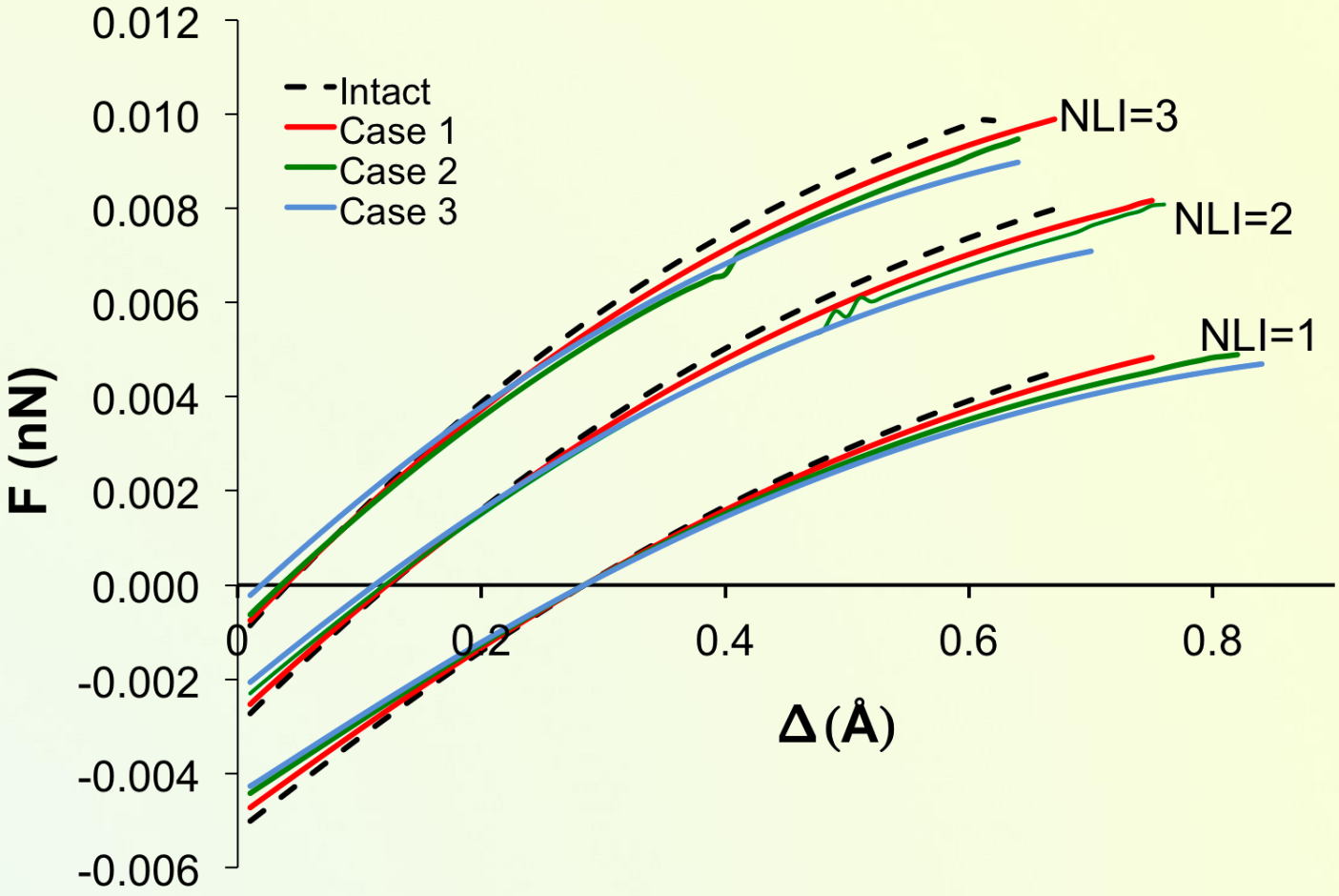


Case 3
Two adjacent nodes removal



Bidimensional Model - with defects

Flaw-tolerance study

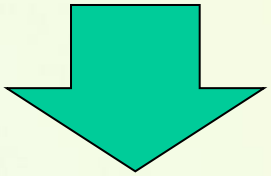


For all given NLI: case 3 is the worst, it is due to:

- *number of removed links (respect to case 1)*
- *position of removed node (respect to case 2)*

Remarks

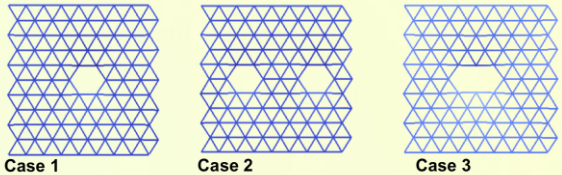
- **The role of NLI is important to characterize the network and has a considerable influence on the load carrying capacity of the system**
- **Strain localization phenomena have been observed in discrete systems with nonlocal links**



Study of force level and distribution across the elements of the discrete system

Bidimensional Model - with defects

Forces distribution in a defective 31 x 31 Å sample

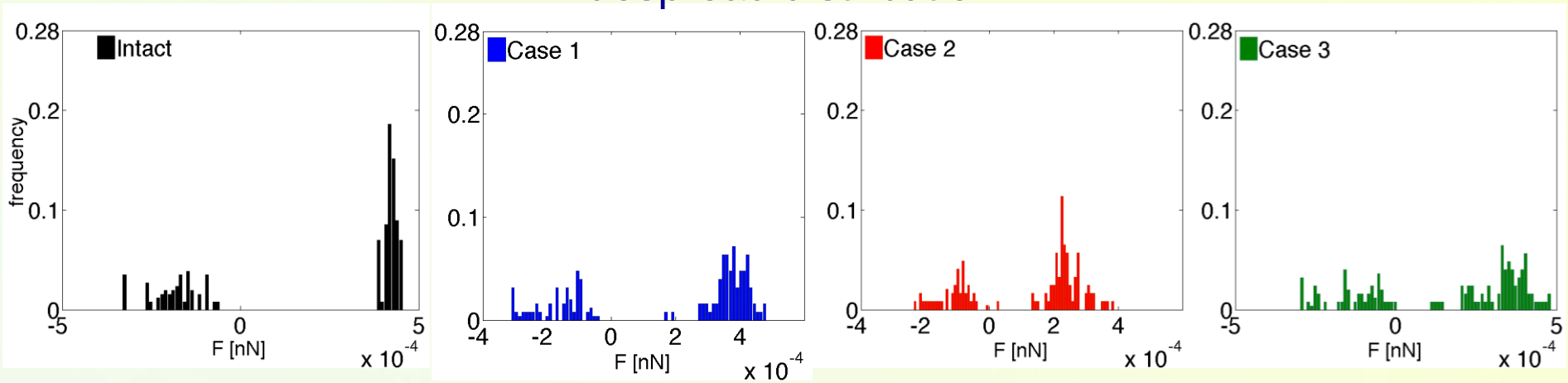


Case 1
One node removal

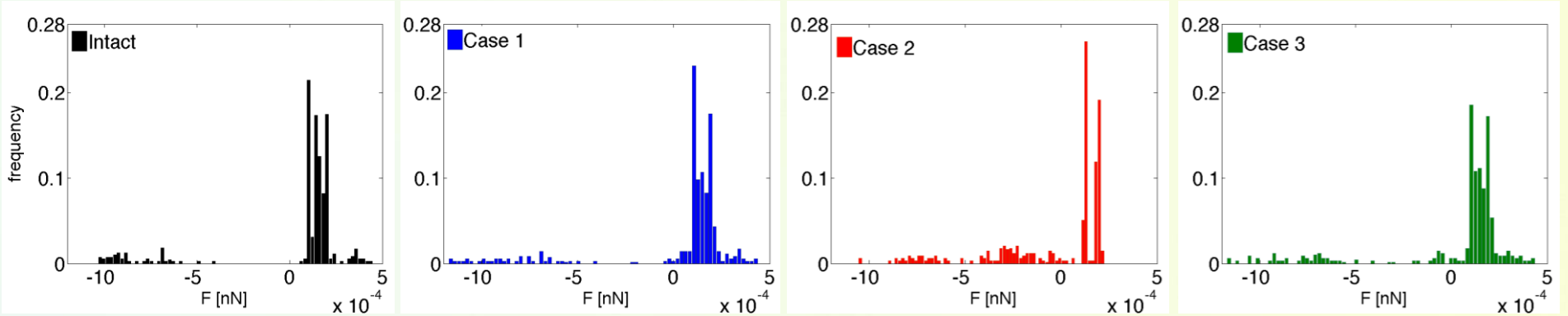
Case 2
Two non adjacent nodes removal

Case 3
Two adjacent nodes removal

NLI = 1 Widespread distribution



NLI = 3 Lower values of forces



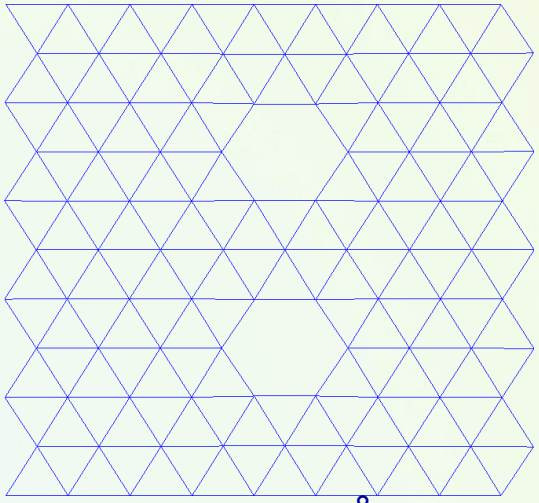
Bidimensional Model - with defects

Forces distribution for different sample dimensions

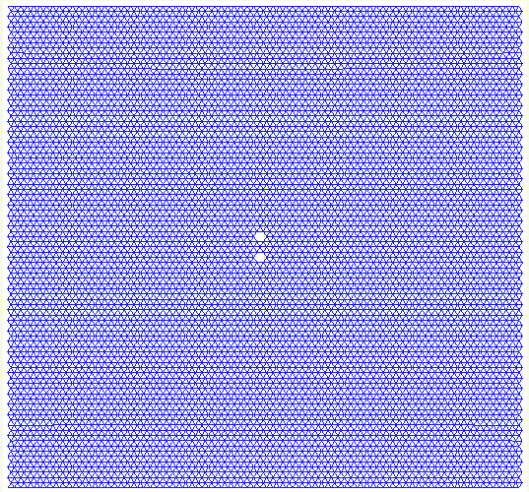
	NLI=3	
L [Å]	n. links	F_{max} [nN]
31	1216	0.776E-04
93	13568	3.849E-04
297	129316	117.57E-04

Results:

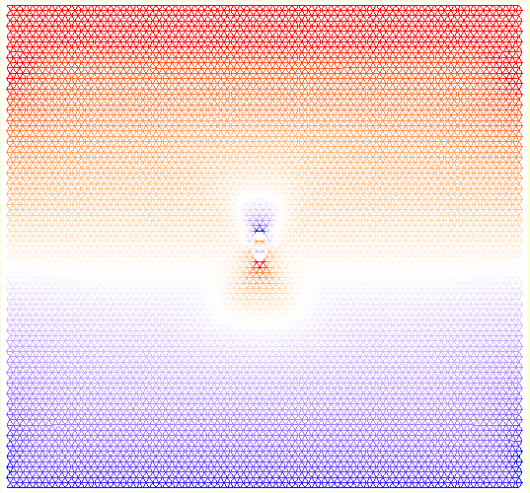
- Peak frequency is reached for lower force values
- More broad force distribution with the increase of dimensions



31 x 31 Å

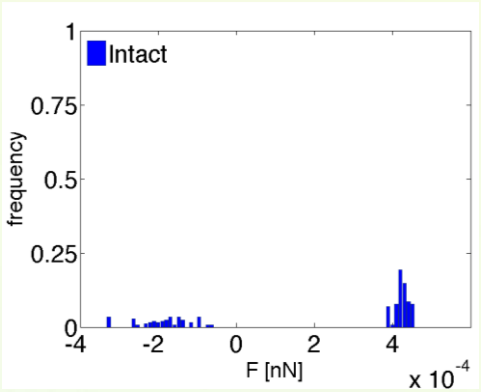


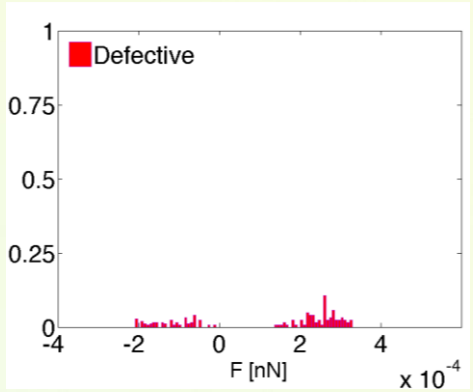
279 x 279 Å



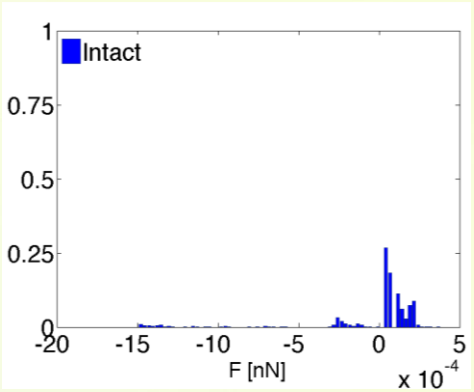
Bidimensional Model - with defects

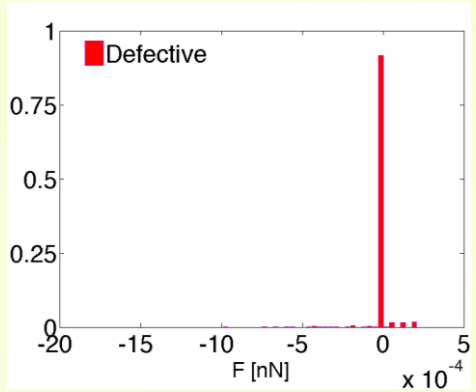
31 x 31 Å – NLI=1





31 x 31 Å – NLI=3

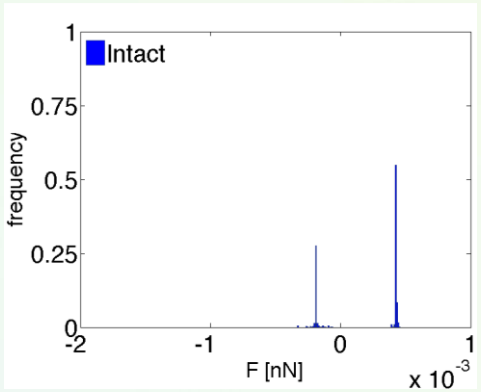


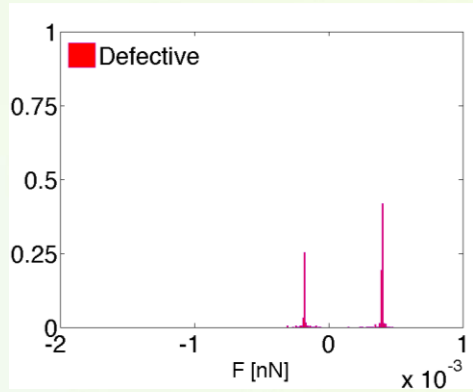


dim. increase = less boundary effects (force more concentrated)

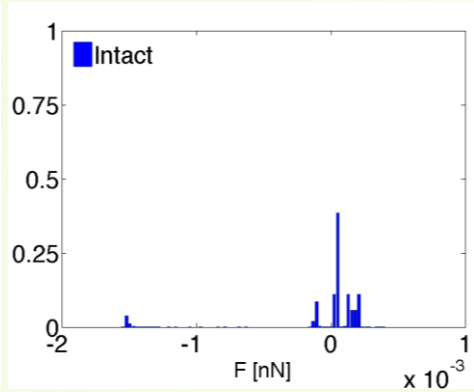
bigger sample is more flaw-tolerant

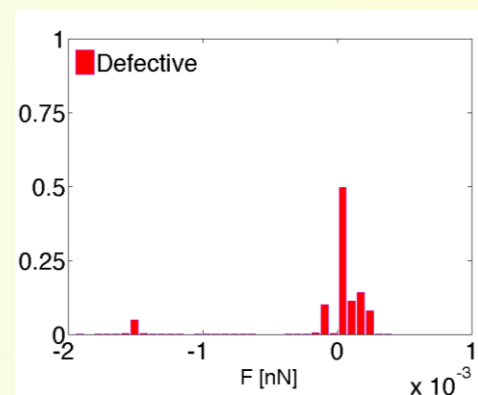
279 x 279 Å – NLI=1





279 x 279 Å – NLI=3



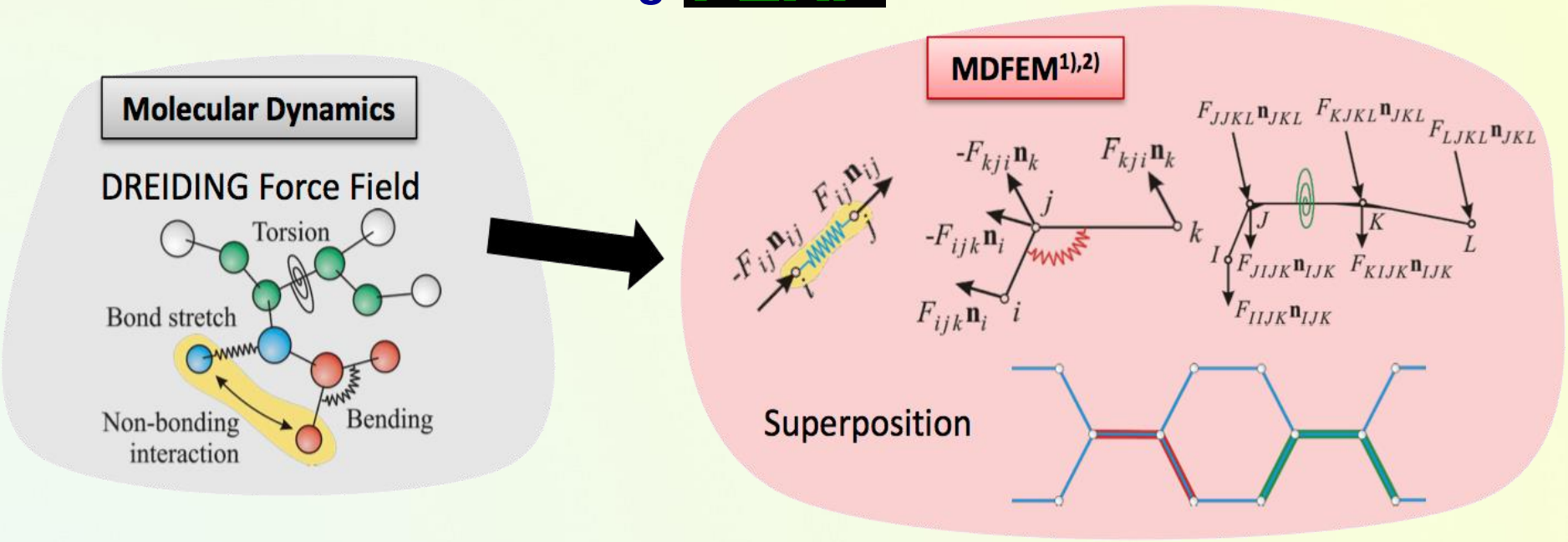


Work in progress – model enrichment

Model completion introducing potential for bonded interactions due to physical links between molecules.

4 nodes element DREIDING (b) + vdW (nb)

MDFEM simulation using FEAP



L. Nasdala et al. (2005) Comp. Mat. Sci. 33: 443-458

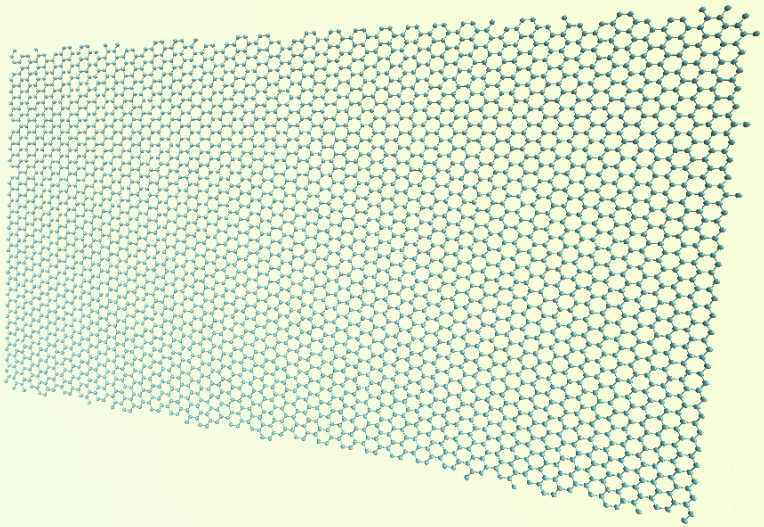
Work in progress – MD model

- ✓ **Pure MD simulation with LAMMPS Molecular Simulator***
- ✓ **AIREBO** (complete) potential considering pairing energy between -C- atoms**

56 atoms
242 links } **sample box**

'p p f' boundary conditions

1.1E+06 total steps @ 5E-01 fs



* S. J. Plimpton (1995), J Comp Phys, 117: 1-19

** D.W. Brenner (2002) J. Phys. Condens. Matter 14:783–802

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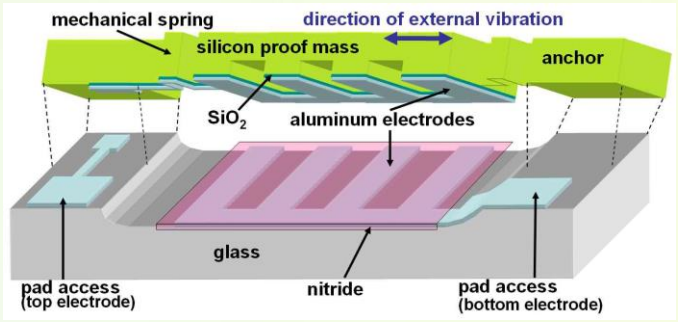
Acknowledgements

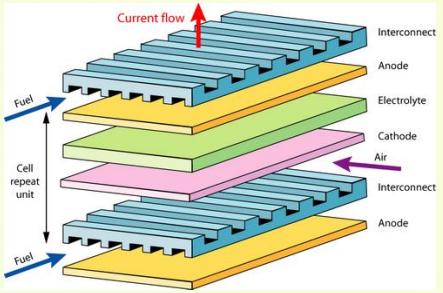
FIRB Future in Research 2010

Structural mechanics models for renewable energy applications

Photovoltaics (POLITO)



MEMS for wind energy harvesting (UNISAL)



Solid oxide fuel cells (UNITN)