COLLOQUIUM 575

CONTACT MECHANICS AND COUPLED PROBLEMS IN SURFACE PHENOMENA

30 March - 2 April 2015, IMT Institute for Advanced Studies Lucca, Italy

Book of Abstracts

Preface

Contact mechanics is a fundamental discipline of the engineering sciences and plays a major role in the understanding of a large number of physical phenomena related to surface interactions between solids. Contact problems are essential for adhesion, friction, capillarity, hydrophobicity, lubrication, thermal and electric conduction across interfaces, control of frictionally-induced vibrations and wear. Moreover, applications cover a large number of scales and range from natural systems like geological faults, engineering systems such as structural adhesives, protective coatings, seals, brakes, clutches, down to micro- or nano-devices used in electro-mechanical systems, and to biological materials and bio-inspired interfaces. In this very active research area, progresses have been made as far as the mathematical description of unilateral contact problems, the uniqueness of their solution and the development of computational methods are concerned.

The purpose of this EUROMECH colloquium featuring 43 invited lectures is to provide an exploratory multi-disciplinary workshop where engineers, mathematicians and physicists can meet and discuss about the latest trends in mathematical modelling, computational methods and experimental research on coupled problems in surface phenomena, a topic still largely unchallenged due to its inherent complexity. In particular, areas of interest regard the solution of contact problems with smooth or rough boundaries in the presence of multiple fields (mechanical, thermal, electro-magnetic, hygrometric fields, chemical reactions, etc.). These multi-physics problems are important in geomechanics and civil engineering (chemo-mechanical coupling in meso-scale models of soil and concrete; fluid-structure interaction at structural interfaces; thermo-hygro-mechanical coupling at interfaces for building materials; contemporaneous presence of fracture and contact at interfaces), in materials for energy applications (thermo-chemo-mechanical coupling at interfaces in solid oxide fuel cells; thermo-electro-mechanical coupling at cracks in solar cells), in bio-mechanics (effect of humidity on contact and wear of hard tissues), in mechanical engineering (coupled tangential and normal contact problems), electronics (thermo-mechanical response of bi-material interfaces in MEMS; sealing of bi-material interfaces), and in a wide range of other surface phenomena.

Lucca, 10/03/2015

Marco Paggi and David Hills
Colloquium Chairmen

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Euromech Colloquium 575 – Programme

Sunday 29 March, 2015

	17:00	20:00	***Welcome cocktail, visit to the Puccini museum house & registration***

Monday 30 March, 2015

	08:30	09:10	***Registration***	
	09:10	09:30	***Opening addresses***	
Session 1 Chairman: D.A. Hills	09:30	10:00	J.R. Barber	Nominally static frictional contacts under periodic loading
	10:00	10:30	F.M. Borodich	Simulations of tribological phenomena using a structural block with nano and micro levels of roughness
	10:30	11:00	M. Cocou	A dynamic viscoelastic problem with slip dependent interfacial friction
	11:00	11:30	*Coffee break*	
Session 2 Chairman: J.R. Barber	11:30	12:00	D. Bigoni	Flutter and friction
	12:00	12:30	A. Rigazzi	Influence of roughness on elastostatic friction for nearly incompressive solids - A Finite Element Method approach
	12:30	12:45	A. Papangelo	On the Griffin model of underplatform frictional dampers with varying normal load
	12:45	13:00	M. Ciavarella	A Griffith model of friction in the transition from stick to slip
	13:00	14:30	*Lunch*	
Session 3 Chairman: D. Bigoni	14:30	15:00	M. Scaraggi	Friction and universal contact area law for randomly rough viscoelastic contacts
	15:00	15:30	A.K. Abramyan	Destruction of thin films with a damaged adhesive substrate as a result of waves localization
	15:30	16:00	F.M. Borodich	Problems of adhesive contact interactions between prestressed biological cells
	16:00	16:30	*Coffee break*	
Session 4 Chairman: F. Borodich	16:30	17:00	S. Lee	Tailoring the lubricity of mucin film at a hydrophobic interface
	17:00	17:30	J. Vázquez	A new procedure for the solution of plane contacts
	17:30	18:00	M. Petrzhik	Characterization of functional surfaces of advanced materials and coatings by local mechanical contact testing

Tuesday 31 March, 2015

Session 1 Chairman: D. Nelias	09:00	09:30	Q.-C. He	Determination of the effective interfacial conditions for Stokes flow on a rough solid surface
	09:30	10:00	J. Haslinger	Stokes problem with threshold slip boundary conditions. Part I: theory
	10:00	10:30	R. Kucera	Stokes problem with threshold slip boundary conditions. Part II: hydrophobic surfaces modelling
	10:30	11:00	*Coffee break*	
Session 2 Chairman: Q.-C. He	11:00	11:30	V. Yastrebov	A numerical study of the contact between rough surfaces: mechanical and transport phenomena at small scales
	11:30	12:00	E. Bertocchi	Analytical evaluation of the peak contact pressure in a rectangular elastomeric seal with rounded edges
	12:00	12:30	A. Wirtz	A combined finite element framework for contact and fluid-structure interaction
	12:30	13:00	A. Konyukhov	Computational contact mechanics methods for the finite cell method
	13:00	14:30	*Lunch*	
	14:30	19:30	***Excursion to Pisa***	

Wednesday 1 April, 2015

Session 1 Chairman: C. Putignano	09:00	09:30	D.A. Hills	A framework for quantification of fretting fatigue
	09:30	10:00	V.L. Popov	On gross slip wear of an axisymmetric punch using MDR
	10:00	10:30	P. Farah	An implicit finite wear contact formulation based on dual mortar methods
	10:30	11:00	*Coffee break*	
Session 2 Chairman: V.L. Popov	11:00	11:30	G. Del Piero	A single state variable model for adhesive interfaces
	11:30	12:00	I. Berardone	Electro-mechanical coupling in cracked Silicon solar cells embedded in photovoltaic modules: experiments and simulations
	12:00	12:30	L. Lanzoni	On the problem of a Timoshenko beam bonded to an elastic half-plane
	12:30	13:00	N. Menga	Adhesive elastic periodic contacts: the role of interfacial friction and slab thickness
	13:00	14:00	*Lunch*	
Session 3 Chairman: G. Del Piero	14:00	14:30	C. Putignano	Viscoelastic contact problems: challenges and recent advancements
	14:30	15:00	D. Nelias	Viscoelastic rolling/sliding contact problem with an heterogeneous material
	15:00	15:30	H.-P. Yin	Numerical simulations of the frictionless contact between the rough surfaces of two elastic or viscoelastic bodies
	15:30	16:00	I. Goryacheva	Sliding contact of a spherical indenter and a viscoelastic base with molecular adhesion
	16:00	16:30	*Coffee break*	
Session 4 Chairman: I. Goryacheva	16:30	17:00	L. Rodriguez-Tembleque	Boundary element solution of contact problems in the presence of electric fields
	17:00	17:30	E. Torskaya	Study of roughness effect on elastic indentation of coated bodies
	17:30	18:00	P. Gourgiotis	The contact problem of a rigid flat punch indenting a couple-stress thermoelastic half-space
	18:00	18:30	K. Houanoh	Numerical study of the frictionless contact problem between thermoelastic wavy surfaces
	20:00		***Social dinner***	

Thursday 2 April, 2015

Session 1 Chairman: I. Argatov	08:30	09:00	G. Anciaux	Influence of plasticity on the real contact area during normal loading of rough surfaces
	09:00	09:30	M. Paggi	Optimization algorithms for the solution of the frictionless normal contact between rough surfaces
	09:30	10:00	T. Chaise	Influence of porosity content on homogenized mechanical properties
	10:00	10:30	E. Olsson	Micromechanical investigation of fracture of cold compacted powder
	10:30	11:00	*Coffee break*	
Session 2 Chairman: M. Paggi	11:00	11:30	I. Argatov	A coupled impact problem for articular cartilage: Phenomenological modeling of damage in a biological tissue under dynamic loading
	11:30	12:00	D. Misseroni	Experimental validation of an asymptotic model to predict crack trajectories influenced by voids
	12:00	12:30	C. Borri	Multiscale characterization of complex surfaces: anti-reflective coatings, hydrophobic surfaces and fibrillar interfaces
	12:30	13:00	M. Giacopini	Preliminary investigation of the cavitation damage in the conrod big end bearing of a high performance engine via a mass-conserving complementarity algorithm
	13:00		***Closing address and lunch box***	

Monday, March 30, 2015

First session

9:30-11:00

Nominally static frictional contacts under periodic loading

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Keywords: Fretting, microslip, shakedown, frictional damping.

Nominally static contacts such as bolted or shrink-fit joints typically experience regions of microslip when subjected to oscillatory loading. This results in energy dissipation, reflected as apparent hysteretic damping of the system, and also may cause the initiation and propagation of fretting fatigue cracks. Early theoretical studies of periodic tangential loading of the Hertzian contact problem by Cattaneo [1] and Mindlin [2] were confirmed experimentally by Johnson [3], who identified signs of fretting damage in the slip annulus predicted by the theory. When both normal and tangential forces vary in time, the energy dissipation is very sensitive to the relative phase of the oscillatory components, being greatest when they are out of phase [4].

For many years, tribologists assumed that Melan's theorem in plasticity could be extended to frictional systems — i.e. that if there exists a state of residual stress associated with frictional slip that is sufficient to prevent periodic slip in the steady state, then the system will shake down, regardless of the initial conditions. However, we now know that this is true only if there is no coupling between the normal and tangential loading problems, as will arise notably in the case where contact occurs on a symmetry plane [5]. We have also recently established that for uncoupled systems, the dissipation per cycle above the shakedown limit is independent of initial conditions [6].

For all other cases, periodic loading scenarios can be devised such that shakedown occurs for some initial conditions and not for others. The initial condition here might be determined by the assembly protocol — e.g. the order in which a set of bolts is tightened — or by the exact loading path before the steady cycle is attained. This non-uniqueness of the steady state persists at load amplitudes above the shakedown limit, in which case there is always some dissipation, but the dissipation per cycle (and hence both the effective damping and the susceptibility to fretting damage) may depend on the initial conditions. This also implies that fretting fatigue experiments need to follow a well-defined assembly protocol if reproducible results are to be obtained. For two-dimensional systems, a criterion for unconditional shakedown can be established, based on the properties of the contact stiffness matrix [7].

Dependence on initial conditions or loading history implies that the frictional system posesses 'memory', which we postulate resides in the locked-in slip displacements at regions that are not at a given time slipping. If all nodes slip at least once during the cycle (but not all at once, which would constitute 'gross slip' or sliding), the memory must be periodically exchanged between nodes and this generally leads to a degradation of memory and hence an asymptotic approach to a unique steady state. However, systems can be identified [typically involving significant normal-tangential coupling] in which there exist several such distinct steady-states, some of which are stable (attractors) and some unstable. If on the other hand there exists a region that never slips during the steady state [the permanent stick zone] then there will exist a real infinity of possible steady states associated with variation in locked-in displacements in this region.

With sufficient clamping force, 'complete' contacts (i.e. those in which the contact area is independent of the normal load) can theoretically be prevented from slipping, but on the microscale, all contacts are incomplete because of surface roughness and some microslip is inevitable. In this case, the local energy dissipation density can be estimated from relatively coarse-scale roughness models, based on a solution of the corresponding 'full stick' problem [8].

References

[1] Cattaneo C 1938 Sul contatto di due corpi elastici: distribuzione locale degli sforzi *Rendiconti dell'Accademia Nazionale dei Lincei* **27** 342–348 434–436 474–478 (In Italian)

[2] Mindlin R D and Deresiewicz H 1953 Elastic spheres in contact under varying oblique forces *ASME Journal of Applied Mechanics* **75** 327–344

[3] Johnson K L 1955 Surface interaction between elastically loaded bodies under tangential forces *Proceedings of the Royal Society of London* **A230** 531–548

[4] Yong Hoon Jang and J.R.Barber, Effect of phase on the frictional dissipation in systems subjected to oscillatory loads, *European Journal of Mechanics A/Solids*, Vol.30 (2011), pp.269–274.

[5] Klarbring A Ciavarella M and Barber J R 2007 Shakedown in elastic contact problems with Coulomb friction *International Journal of Solids and Structures* **44** 8355–65

[6] L-E. Andersson, J. R. Barber and A. R. S. Ponter, Existence and uniqueness of attractors in frictional systems with uncoupled tangential displacements and normal tractions, *International Journal of Solids and Structures*, Vol. 51 (2014), pp. 3710–3714.

[7] Ahn Y J Bertocchi E and Barber J R 2008 Shakedown of coupled two-dimensional discrete frictional systems *Journal of the Mechanics and Physics of Solids* **56** 3433– 40

[8] Putignano C Ciavarella M and Barber J R 2011 Frictional energy dissipation in contact of nominally flat rough surfaces under harmonically varying loads *Journal of the Mechanics and Physics of Solids* **59** 2442–54

Simulations of tribological phenomena using a structural block with nano and micro levels of roughness

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Keywords: Friction, adhesion, roughness, hierarchical model, multilevel structure

Structural models of rough surfaces, i.e. models whose structure is described explicitly or in a statistical way, are widely used in tribology. A very popular model is a statistical structural model that presents all surface protuberances as elastic spheres having different heights but the same radius of curvature of summits. The model was originally proposed by Zhuravlev [1] and it was later developed by Kragelsky [2] and by Greenwood and Williamson [3].

In the framework of this ZGW model, it was shown for many functions of distributions of the heights of the surface protuberances, the true contact area is approximately proportional to the external compressing force. Assuming the frictional force is caused by molecular interaction between atoms and proportional to the true contact area S, Zhuravlev gave an explanation of the Amontons-Kotelnikov law of friction $F_f = \mu P$ where F_f is the friction force, μ is the coefficient of friction (COF), and P is the normal load (in fact, Amontons (1699) gave a verbal formulation of the law as three relations observed for optical lens polishing process, while Kotelnikov (1774) introduced the notion of coefficient of friction μ and presented the law as a formula). However, the ZGW model was later criticized. The main criticism was that the radii of the asperity spheres depend on the scale of consideration; hence the model is scale dependent. In addition, the molecular adhesion between rubbing surfaces was neglected and the hierarchical character of roughness discovered by Archard [4] was not reflected in the model. Finally it did not consider the possible interplay between asperities during the shift of the surfaces.

We present a structural model of a nominally flat block having hierarchical structure. The proposed model is used for numerical simulations of frictional motion of the block sliding along a rough surface. Because both surfaces are rough, the true contact area changes during sliding and, therefore, the friction is not constant. The goal is to develop such a model of roughness that has experimentally observed tribological characteristics and it is free from the above drawbacks of the ZGW model. The main features of the proposed model are the following.

(i) The model reflects the hierarchical structure of the rough surfaces (this idea was discussed in a number of papers); the employment of two level (nano and micro) measurements of the roughness allows us to reflect the complex structure of real surfaces better than the conventional models.

(ii) The model takes into account both elastic contact and the molecular adhesion between the counterparts; the adhesion is described using an approach that is similar to the Maugis step-function description of the adhesive zone.

(iii) It is assumed that the true COF is constant; we follow the Derjaguin idea [5] of distinguishing between the true COF μ_t and the apparent COF μ_a where and p_0 is the specific attractive force,

$$\mu_t = \frac{F_f}{P + Sp_0} \quad \text{and} \quad \mu_a = \frac{F_f}{P}.$$

(iv) The parameters of the first level of the model (the nano-scale roughness) are extracted from the AFM measurements of the surface roughness, while the parameters of the next level of the model (the micro-scale roughness) are extracted from the data obtained by the measuring of the same surface by a stylus; the elastic properties of the third level are the same as the elastic properties of the bulk material.

(v) The surface roughness is represented not only by rms heights, and the size of asperities but also the average distance (λ) between asperities.

(vi) The average width of asperities at each level is calculated according to the corresponding experimental Abbott bearing curve.

(vii) The asperities of each level are represented not as spheres but rather as elastic flat-ended rods. This idea was borrowed from Kragelsy's modification of the ZGW model [2] and from Borodich & Onishchenko [6, 7] multilevel profile model.

Often the analysis of contact problems for rough surfaces is based on a multi-asperity approach. This means that first the problem is solved for an asperity and then the solution is used to represent the multi-asperity contact assuming that there is no influence of an asperity deformation on other asperities. One of the specific features of the proposed model is that vertical deformation of an asperity affects the whole system. The multiscale nature of the rough surface is taken into account by just two (nano and micro) levels of roughness. The model under development takes into account not only the hierarchical structure of roughness but also the adhesive interactions, the deformation of asperities, transfer the deformations between levels of the hierarchical structure and the vertical degree of freedom of the asperities. It is argued that the model can be extended in various ways. For example, the model may be enhanced by introducing horizontal elastic interactions between asperities [6, 8]. By removing some asperities of the structure, one can model the surface wear and other surface phenomena.

Acknowledgements

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References

[1] Zhuravlev, V.A., "On question of theoretical justification of the Amontons-Coulomb law for friction of unlubricated surfaces", Zh. Tekh. Fiz (Journal of Technical Physics) # 10, 1447-1452 (Russian). Engl. transl. Proc. IMechE, Part J: J. Engineering Tribology, 221, 895–898 (2007).

[2] Kragelsky, I.V., "Static friction between two rough surfaces", Bul. USSR Acad. Sci. Div. Tech. Sci., # 10, 1621–1625 (1948).

[3] Greenwood, J.A., Williamson, J.B.P., "The contact of nominally flat surfaces", Proc. R. Soc. Lond. A, 295, 300–319 (1966).

[4] Archard, J.F., "Elastic deformation and the laws of friction", Proc. R. Soc. Lond. A, 243, 190-205 (1957).

[5] Derjaguin, B.V., "Molekulartheorie der äußeren Reibung", Z. Physik, 88, 661-164 (1934).

[6] Borodich, F.M., Onishchenko, D.A., "Fractal roughness in contact and friction problems (the simplest models)", J. Friction and Wear, 14, 14–19 (1993).

[7] Borodich, F.M. and Onishchenko, D.A. "Similarity and fractality in the modelling of roughness by a multilevel profile with hierarchical structure", Int. J. Solids and Struct, 36(17), 2585-2612 (1999).

[8] Schmittbuhl, J., Vilotte, J.-P., Roux, S., Velocity weakening friction: A renormalization approach, J. Geophys. Res., 101, 13911–13917 (1996).

A dynamic viscoelastic problem with slip dependent interfacial friction

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Keywords: Dynamic problems, viscoelasticity, slip depending friction, variational analysis.

This work is concerned with the extension of the recent results presented in [2] for some non-smooth dynamic frictional contact problems to the case of a coefficient of friction depending on the slip velocity [3]. This class of problems constitutes a unified approach to study some interface models, including relaxed unilateral contact conditions, adhesion, and pointwise friction.

Dynamic viscoelastic contact problems with nonlocal friction laws were considered in [9, 6, 1, 4] and the corresponding problems with normal compliance laws have been analyzed in [7, 8, 10]. Dynamic frictionless problems with adhesion have been studied by several authors, see, e.g. [12] and references therein, and dynamic viscoelastic problems coupling unilateral contact, recoverable adhesion and nonlocal friction have been investigated in [5].

Based on new variational formulations and some approximation results, the approach proposed in [11] in the elastostatic case is extended to the dynamic contact between two viscoelastic bodies of Kelvin-Voigt type. Finally, several examples are presented to illustrate the generality of the proposed interfacial model.

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References

[1] Cocou, M., "Existence of solutions of a dynamic Signorini's problem with nonlocal friction in viscoelasticity", Z. Angew. Math. Phys., 53, 1099-1109 (2002).

[2] Cocou, M., "A class of dynamic contact problems with Coulomb friction in viscoelasticity", Nonlinear Analysis RWA, 22, 508-519 (2015).

[3] Cocou, M., "A variational analysis of a class of dynamic problems with slip dependent friction", Annals of the University of Bucharest (mathematical series), to appear.

[4] Cocou, M., Scarella, G., "Analysis of a dynamic unilateral contact problem for a cracked viscoelastic body", Z. Angew. Math. Phys., 57, 523-546 (2006).

[5] Cocou, M., Schryve, M., Raous, M., "A dynamic unilateral contact problem with adhesion and friction in viscoelasticity", Z. Angew. Math. Phys. (ZAMP), 61, 721-743 (2010).

[6] Eck, C., Jarušek, J., Krbec, M., "Unilateral Contact Problems - Variational Methods and Existence Theorems", Chapman & Hall/CRC, Boca Raton (2005).

[7] Kikuchi, N., Oden, J., "Contact Problems in Elasticity : A Study of Variational Inequalities and Finite Element Methods", SIAM Studies in Applied Mathematics, SIAM, Philadelphia (1988).

[8] Kuttler, K.L., ”Dynamic friction contact problems for general normal and friction laws”, Nonlinear Analysis TMA, 28, 559-575 (1997).

[9] Kuttler, K.L., Shillor, M., ”Dynamic bilateral contact with discontinuous friction coefficient”, Nonlinear Analysis TMA, 45, 309-327 (2001).

[10] Migórski, S., Ochal, A., Sofonea, M., ”Nonlinear Inclusions and Hemivariational Inequalities”, Springer, New York (2013).

[11] Rabier, P.J., Savin, O.V., ”Fixed points of multi-valued maps and static Coulomb friction problems”, J. Elasticity, 58, 155-176 (2000).

[12] Sofonea, M., Han, W., Shillor, M., ”Analysis and Approximation of Contact Problems with Adhesion or Damage”, Chapman & Hall/CRC, Boca Raton (2006).

Monday, March 30, 2015

Second Session

11:30-13:00

Flutter and friction

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Keywords: Hopf bifurcation, Coulomb friction, Flutter Instability.

The first experimental evidence that Coulomb friction can induce flutter and divergence instability was given by Bigoni and Noselli [1], working on a special version of the Ziegler pendulum [2]. Experiments have fully confirmed the theoretical expectations and the instabilities have been proven to be robust to several perturbations in the experimental set-up [3].

Bigoni and Noselli were unable to induce flutter more complicated systems that the Ziegler pendulum, such as for instance the Beck's column [4]. The Bigoni and Noselli's apparatus has therefore been completely redesigned in order to provide a more flexible testing apparatus. This new design has allowed us to experimentally investigate the behavior of the Beck column. Results on this and related problems will be presented in detail.

Acknowledgements

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References

[1] D. Bigoni and G. Noselli Experimental evidence of flutter and divergence instabilities induced by dry friction. *J. Mech. Phys. Solids*, **59**, 2208–2226 (2011).
[2] Ziegler, H. Linear elastic stability. A critical analysis of methods. *Z. Angew. Math. Phys.*, **4**, 89-121 (1953).
[3] D. Bigoni, D. Misseroni, G. Noselli, D. Zaccaria Surprising instabilities of simple elastic structures. In Nonlinear Physical Systems - Spectral Analysis, Stability and Bifurcations, Kirillov, N. and Pelinovsky, D.E. Eds., Wiley, pp. 1-14, 2014; ISBN: 978-1-84821-420-0.
[4] Beck, M. Die Knicklast des einseitig eingespannten, tangential gedruckten Stabes. *Z. Angew. Math. Phys.*, **3**, 225-228 (1952).

Influence of roughness on elastostatic friction for nearly incompressible solids – a Finite Element Method approach

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Keywords: Static friction, roughness, Finite Element Method.

We present the results of our recent work, in which we apply the Finite Element Method in order to numerically study the contact between an elastic smooth cube and several rigid rough surfaces. Our goal is to compute the influence of different surface parameters on the real area of contact and on the elastostatic friction force transferred at the contact boundary.

Every real surface is – at some length scale – rough. As a consequence, contact happening in the real world is never smooth, especially at micro- or nano-scale. The influence of roughness on contact features like the real area of contact and the stress distribution is the subject of different theoretical models [1, 2, 3], but microscopic roughness also affects physical phenomena observed at macroscopic scale, such as static and kinetic friction. Friction models often cite viscosity and plasticity as causes for friction, neglecting or relegating to the second plane effects of purely elastic interactions [4, 5, 6].

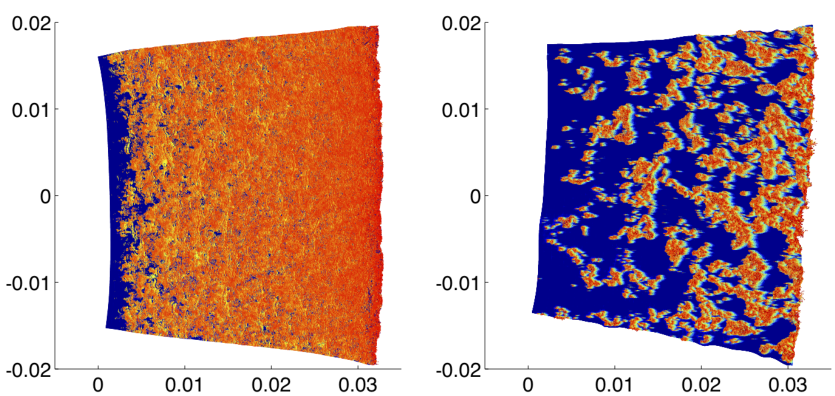

Figure 1: Surface of an elastic cube in contact with two different rough surfaces.

In our study, we apply the FEM at a microscopic length scale, but on a macroscopic domain. We prove that – even in the frame of linear elasticity – non-negligible forces opposed to the tangential sliding of the body can be measured, when computing the correct contact normals and employing a sufficiently dense discretization of the elastic cube.

In order to resolve the strongly non-linear constraints arising from the geometric complexity of the rough surfaces, we employ an SQP-like method, in which the solution of the contact problem is obtained as a sequence of approximations computed with linearized contact constraints.

We focus on the determination of the real area of contact for different loads, and on the prediction of static friction force generated only by the interactions of the rigid asperities with the elastic cube.

To compute the relation between normal force and real area of contact, we apply an incremental load on top of the elastic cube. To measure static friction, we simulate shearing tests: the elastic cube is first pushed onto the rough surface, and then sheared to a configuration in which the elastic shear force is balanced by the resistance force transferred at the contact boundary. The studies are performed on a broad range of surfaces, differing for roughness parameters such as Hurst exponent and root mean square roughness.

We compare our predictions of the contact area to existing rough contact models (BGT, Persson), and those of the friction force to experimental data [7, 8]. Our results show possible limits of applicability and reliability of the rough contact models, and are in good agreement with experimental data.

Finally, we show that the numerical experiments we perform give reliable results only when employing a very dense computational mesh. Thus a contact problem with 45 millions of unknowns needs to be solved. For this reason, the experiments are performed with our massively parallel multigrid based contact solver [9, 10], on machines with up to 2048 processors.

References

[1] J. A. Greenwood and J. B. P. Williamson. Contact of Nominally Flat Surfaces. *Proceedings of the Royal Society of London. Series A, Mathematical and Physical Sciences*, 295(1442):300–319, 1966.

[2] A.W. Bush, R.D. Gibson, and T.R. Thomas. The elastic contact of a rough surface. *Wear*, 35(1):87 – 111, 1975.

[3] B. N. J. Persson. Theory of rubber friction and contact mechanics. *Journal of Chemical Physics*, 115(8):3840–3861, 2001.

[4] W. R. Chang, I. Etsion, and D. B. Bogy. Static friction coefficient model for metallic rough surfaces. *Journal of Tribology*, 110(1):57–63, 01 1988.

[5] H. Matsukawa and H. Fukuyama. Theoretical study of friction: One-dimensional clean surfaces. *Phys. Rev. B*, 49:17286–17292, Jun 1994.

[6] L. Li, I. Etsion, and F. E. Talke. Contact area and static friction of rough surfaces with high plasticity index. *Journal of Tribology*, 132(3):031401–031401, 06 2010.

[7] J. J. Henry, National Cooperative Highway Research Program, National Research Council (U.S.). Transportation Research Board, American Association of State Highway, and Transportation Officials. *Evaluation of Pavement Friction Characteristics*. NCHRP synthesis. National Academy Press, 2000.

[8] J. N. Meegoda. Non-contact skid resistance measurement. Technical report, New Jersey Institute of Technology, 2009.

[9] R. Krause. A nonsmooth multiscale method for solving frictional two-body contact problems in 2d and 3d with multigrid efficiency. *SIAM Journal of Scientific Computing*, 31(2):1399–1423, January 2009.

[10] R. Krause and C. Mohr. Level set based multi-scale methods for large deformation contact problems. *Appl. Numer. Math.*, 61(4):428–442, April 2011.

On the Griffin model of underplatform frictional dampers with varying normal load

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Keywords: Extended Griffin model, contact stiffness, varying normal load.

The resonant response of a turbine airfoil is generally attenuated, especially in hot parts, with the use of "Coulomb dampers", also called "cottage-roof dampers", "underplatform dampers", or "friction dampers", which are loaded by centrifugal forces against the underside of two adiacent blades. In a classical study, Griffin [1] proposed a very simple model involving a single mass representing the blade, attached to the wall by a single spring (of stiffness k) in parallel with a Coulomb damper having a spring of stiffness k_d (which could alternatively represent a contact stiffness at the level of asperities) and closed by a constant force N. This model is simple enough for an harmonic balance approximate solution to be developed in closed form, permitting to find the optimal normal load which corresponds to the optimal mass of the damper under the centrifugal force. This optimal load corresponds to a “bounded response” regime, mid-way between the resonant frequencies of the free- and that of the stuck-interface. Later studies have concentrated on more realistic models, including more degrees of freedom, variation of normal load in the damper, possible rolling motions of the damper, extended contact, etc. However, the simplicity is lost, it is not possible to gather information on the optimal configuration as in the basic Griffin findings. One may wonder, for example, if the influence of normal load variation would change the conclusions qualitatively. The effect of normal load variation in simple single degree of freedom model has been studied only by Jang and Barber [2] but in the quasi-static limit, suggesting a strong influence of the phase between normal and tangential loads. We have therefore recently studied the "Den Hartog" single degree of freedom model [3], both with a piecewise analytical solution joining all the slip phases, and with a full time-marching Newmark algorithm [4]. We found that dissipation can be much higher for quadrature (and generally out-of-phase) loading, as noticed also by Jang and Barber for the quasi-static limit. However, this does not correspond to higher "damping" of vibrations, but rather the opposite: indeed, for in-phase loads, variation of normal loads can lead to a large reduction of both dissipation and displacement, not expected from the quasi-static prediction. Here, we extend these results to the full Griffin model.

References

[1] Griffin J. H., "Friction Damping of Resonant Stresses in Gas Turbine Engine Airfoils", *J. Eng. Gas Turbines Power* **102** (2), 329-333 (1980).
[2] Jang Y. H. and Barber J.R., "Frictional energy dissipation in materials containing cracks", *Journal of the Mechanics and Physics of Solids*, **59**(3), 583--594 (2011).
[3] Den Hartog J. P., "Forced vibrations with combined Coulomb and viscous friction", *Transaction of the ASME*, **53**, 107-115 (1931).
[4] Newmark N. M., "A Method of Computation for Structural Dynamics", *Journal of the Engineering Mechanics Division*, **85 (**3), 67-94, (1959).

A Griffith model of friction in the transition from stick to slip

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Keywords: Friction, Fracture Mechanics, Cattaneo-Mindlin problem

The transition from stick to slip in the classical Coulomb friction for Hertzian contact has been studied by Cattaneo and Mindlin. The present author has also generalized their method of superposing the full sliding component of shear with a corrective distribution in the stick region, for general geometries. Faults models in geophysics and recent high-speed highresolution measurement of the real contact area and strain fields for nominally flat rough interfaces at macroscopic but laboratory scale, all have suggested an alternative to Coulomb law, given by the classical Griffith fracture mechanics singular solutions of shear cracks. Here, we develop the extension of the CM solution for such a friction model, maintaining the Hertzian solution for the normal pressures, and Coulomb law in the slip region. The model departs from the standard Cattaneo-Mindlin solution for high toughness, showing an increased size of the stick zone relative to the contact area. Further, a sudden transition to slip when the stick region occurs at a critical size, which corresponds to the pull-off contact size of the JKR solution. An apparent static friction coefficient just before full sliding can be defined, and is higher than the sliding friction coefficient. Following experimental evidence as well as theoretical reasoning, a pressure-dependent friction "toughness" is also introduced, due to the sparseness of the contact area. Qualitative agreement with Fineberg's group experiments in PMMA exists: if the stick-slip boundary quasi-static prediction corresponds to the arrest of their slip "precursors", then the rapid collapse to global sliding when the precursors arrest front has reached about half the interface, may be interpreted here to correspond to the "critical" size for the stick zone. Finally, we remark that even the original Cattaneo-Mindlin solution can resemble closely a singular solution, and the extracted apparent strength of the singularity can also be close to a value obtained from the singular solution. Therefore, a final evidence for the Griffith model of friction may require further effort, particularly as the effect of roughness in principle should destroy any effect of such singularities, as it does for adhesion according to the models of Fuller and Tabor.

References

[1]. Adams, G.G., Stick, partial slip and sliding in the plane strain micro contact of two elastic bodies. R. Soc.open sci. 1: 140363 (2014).
[2]. Ben-David, O., Cohen G and Fineberg, J., The Dynamics of the Onset of Frictional Slip, Science: Vol. 330 no. 6001 pp. 211-214 (2010).
[3]. Carpinteri, A., and Paggi. M., Size-scale effects on the friction coefficient. Int J Solids Struct42.9. 2901-2910 (2005).
[4]. Cattaneo, C., 1938., Sul contatto di due corpi elastici: Distribuzione locale degli sforzi. Reconditi dell'Accademia nazionale dei Lincei, 27, 342-248, 434436, 474-478 (1938).
[5]. Ciavarella M., "The generalized Cattaneo partial slip plane contact problem. I-Theory Int. J. Solids Struct., Vol. 35/18, pp. 2349-2362 (1998).
[6]. Ciavarella, M., Tangential loading of general three-dimensional contacts. ASME J. Appl. Mech. 65, 998—1003 (1998).
[7]. Dini, D., Hills D.A., Bounded asymptotic solutions for incomplete contacts in partial slip, Int J Solids StructVolume: 41, Issue: 24-25, 70497062 (2004)

[8]. Kammer, D.S. Radiguet, M., Ampuero, JP, Molinari, JF, Linear elastic fracture mechanics predicts the propagation distance of frictional slip, http://arxiv.org/pdf/1408.4413.pdf (2014).
[9]. Svetlizky I, and Fineberg, J., Classical shear cracks drive the onset of dry frictional motion, Nature 509, 205—208 (2014).

Monday, March 30, 2015

Third Session

14:30-16:00

Friction and universal contact area law for randomly rough viscoelastic contacts

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Keywords: rubber friction, contact mechanics, roughness.

We present accurate numerical results[1] for the friction force and the contact area for a viscoelastic solid (rubber) in sliding contact with hard, randomly rough substrates. The rough surfaces are self-affine fractal with roughness over several decades in length scales. We calculate the contribution to the friction from the pulsating deformations induced by the substrate asperities. We also calculate how the area of real contact, $A(v,p)$, depends on the sliding speed v and on the nominal contact pressure p, and we show how the contact area for any sliding speed can be obtained from a universal master curve $A(p)$, see Fig. 1. The numerical results are found to be in good agreement with the predictions of an analytical contact mechanics theory.

Acknowledgements
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References

[1] Scaraggi, M., Persson, B.N.J., "Friction and universal contact area law for randomly rough viscoelastic contacts", J. Phys. Cond. Mat., accepted (2015).

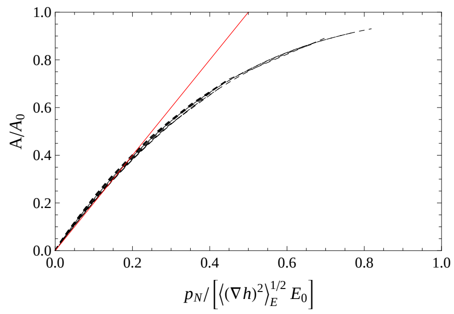

Figure 1: The numerically-calculated nominal contact area A/A_0 as a function of the reduced contact pressure, for several values of roughness cut-off frequencies and sliding velocities. All the curves appear superposed to an unique mastercurve. The red line has a slope of 2.

Destruction of thin films with a damaged adhesive substrate as a result of waves localization

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Keywords: thin film, wave localization, damage substrate.

Some types of a laminated safety glass consists of an interlayer surrounded by two thin films (see in [1]). An optimal level of adhesion is then required for the safety glass to absorb enough of the impact energy to prevent projectile penetration and delamination. Such considerations have motivated us to study the behavior of such structures undergoing periodic strike like loading and introduce the mathematical model of the structure behavior. The structure under consideration is modeled as a string (cut from a thin film), an elastic foundation of which is assumed to be imperfect, and which coefficient depends on a damage function of a substrate.The imperfection of an elastic foundation is modeled by a damage function for which the evolution equation is introduced.The governing equation is as follows:

$$\gamma u_{xx} - K(n)u - \rho_0 u_{tt} = A\delta_\epsilon(x - x_0)\sum_{j=0}^{M}\delta(t - j\Delta t), \quad x \in (-\infty, +\infty), \quad t > 0 \qquad (1)$$

where u is a displacement, A is an amplitude of an external force, Δt is a time step for the strike, δ_ϵ is a smoothed δ function.The quantity K depends on n via the following relations

$$K(n) = \mu(n)G(n), \quad \mu(n) = \frac{k_0}{k_0 + G(n)} \qquad (2)$$

where $k_0 > 0$ is a constant, $G(n) = G_0(1 - n), \quad 0 \le n \le 1$; $G(n) = 0, \quad n > 1$, Here G_0 is a constant. The time evolution of the damage function $n(x, t)$ is defined by the differential equation

$$\frac{\partial n}{\partial t} = \beta H(\mu(n)|u| - \Delta)(1 - n), \qquad (3)$$

where $\beta, \Delta > 0$ are positive constants, and H is the Heaviside step function. we set the following boundary and initial conditions $u(x, t) \to 0 \quad (|x| \to \infty); u(x, 0) = 0, \quad u_t(x, 0) = 0, \quad x \in (-\infty, +\infty); n(x, 0) = n_0(x), \quad x \in (-\infty, +\infty)$ where n_0 is a fast decreasing in $|x|$ smooth function.If $x \in [-h, h]$, we set the Dirichlet boundary condition $u(h, t) = u(-h, t) = 0$. Also we assume that our material reached the full destruction at a point x at the moment t if the damage parameter $n(x, t)$ attain, at this moment, to some critical level n_* close to 1(the elastic foundation coefficient vanished and the delamintion occurs).Let us introduce the front of full destruction $L_F(t)$ by the relation $n_* = n(L_F(t), t)$, where n_* is a critical damage level. The problem (1) for the case when the structure was under harmonic loading was considered in [2].Here we will show that a wave localization is also possible for the non-stationary loading as well. If β is small, then $dn/dt << 1$ and the coefficient K is a function of the slow time $\tau = \beta t$. Therefore,we can use a quasi-stationary approximation by the two-time scales perturbation method. We assume that $\beta << 1$ and $K(n)$ is a function of the slow time $\tau = \beta t$. For each fixed τ, there occurs a Schrodinger operator associated with this problem,

$$\mathcal{H}\Psi = \frac{d^2\Psi}{dx^2} - \Phi(x,\tau)\Psi = E\Psi, \quad \Phi = \gamma^{-1}(K(n(x,\tau)) - \frac{k_0 G_0}{k_0 + G_0}). \tag{4}$$

The eigenfunctions Ψ may be localized ones, when $\Psi = \Psi_j(x,\tau),\ E = \gamma^{-1}(\bar{k}_0 - \rho_0^{-1}\omega_j^2)$, where $j = 1, ..., N(\tau)$, and may belong to the continuous spectrum, when $E = -k^2 \leq 0$ and $\Psi(x,k,\tau)$ have asymptotic $\exp(ikx)$ as $x \to +\infty$ and $a(k)\exp(ikx) + b(k)\exp(-ikx)$ as $x \to -\infty$, where $a(k), b(k)$ are some coefficients.For the continuous spectrum it is useful to introduce the frequency ω by $E(k) = \gamma^{-1}(k_0 - \rho_0\omega^2(k))$. The analysis shows that the following conclusions can be made:for all times there always exists a localized eigenfunction and, thus, $N(\tau) \geq 1$; the mode number N also can depend on τ and it increases in time. One can obtain an asymptotic solution consisting of the two following contributions:

$$u(x,t,\tau) = W(x,t,\tau) + V(x,t,\tau), \tag{5}$$

where

$$V(x,t,\tau) = -\rho_0^{-1}\sum_{j=1}^{N(\tau)} a_j(c\omega_j(0))^{-1}\sin(S_j(t))\Psi_j(x,\tau) \tag{6}$$

$$W(x,t,\tau) = -\rho_0^{-1}\int_0^{\infty} a_k(\tau)\omega(k,0)^{-1}\Psi(x,k,\tau)\sin S(k,t)dk, \tag{7}$$

where

$$a_j(\tau) = \int_{-\infty}^{\infty}\delta_\epsilon\Psi_j(x,\tau)dx, \quad a_k(\tau) = \int_{-\infty}^{\infty}\delta_\epsilon\Psi(x,k,\tau)dx. \tag{8}$$

The asymptotic solutions where obtained for particular types of initial distributions of damaged function n. From them it can be concluded: the second term determine the oscillating character of the front motion, but for $t >> 1$ the wave term is larger and oscillations become smaller and smaller.On the contrary, for $k_0 >> 1$ the wave contribution is small. Using obtained solutions it was possible to calculate the time when the destruction process start and also the time which is necessary for a full destruction.We have two qualitatively different dynamics of the front. If the localized mode amplitude is small with respect to the amplitude of the wave term the dynamics of the front is linear. If the localized mode amplitude has the same order then the wave term amplitude, we have alternation of time intervals,where L_F increases, and time intervals where $L_F(t)$ is a constant. Main effects exhibiting by the suggested model are as follows: 1. There is an incubation period before the destruction process start. 2. A sequence of resonances has been found for a periodic strike like loading. 3. There are possible transitions between destruction types and two types of the front dynamics. 4. Three scenario of $n(x,t)$ behavior has been found: a monotone growth of n; a piecewise like constant growth of n (when n increases during some intervals and n is constant between these growth intervals); no growth of n.

References

[1] Nanshu Lu et al.,Failure by simultaneous grain growth, strain localization, and interface debonding in metal films on polymer substrates,*J.Mater.Res.*, **24**:2,379-385,2009.

[2] A.K.Abramyan et al.,Influence of dynamic processes in a film on damaged development in an adhesive base,*Mechanics of Solids* **47**,5:498-504,2012.

Problems of adhesive contact interactions between prestressed biological cells

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Keywords: Adhesion, biological cell, contact, prestresses, nanomanipulation.

We deal with effects caused by molecular adhesion between biological objects. Cell adhesion is of crucial importance for numerous physiological and pathological processes, including embryonic development, and cancer metastasis, as well as for numerous biotechnological applications [1]. Here we consider problems of adhesive contact interactions between biological cells and other biological materials such that the classic geometrically linear formulation of the Hertz-type contact is applicable, hence the classic JKR (Johnson, Kendall, and Roberts), DMT and Maugis theories may be used. We assume also that the adhesive interaction may be reduced only to attraction between points at the surfaces of the solids and by introduction of the work of adhesion, i.e. the Derjaguin approximation is applicable [2]. Evidently, the present report does not cover all types of biological cells because the structures of biological cells may vary, e.g. we do not consider living cells with molecular brushes. However, we believe that our results can be applied to many types of smooth cells, in particular to red blood cells (RBCs) that have been extensively studied as a relatively simple example of biological cells whose membrane can be represented as composed of a skeleton and a lipid bilayer. The JKR theory of adhesive contact has been widely used as a basis for modelling of various phenomena, in particular biological phenomena such as adhesion of cells, viruses, attachment devices of insects and so on [3–5]. However, the JKR theory was originally developed for linearly elastic isotropic materials, while many biological objects have layered structure, i.e. they are in fact transversely isotropic [2]. In addition, it is known that cell membranes can be considered as presteressed non-linear materials, whose stiffness increases in proportion with the level of the tensile prestress [6].

It is shown that the frictionless JKR model may be generalized to arbitrary convex, blunt axisymmetric bodies, in particular to the case of the punch shape being described by monomial (power-law) punches of an arbitrary degree $d \geq 1$ [7]. The JKR and Boussinesq-Kendall models are particular cases of the problems for monomial punches, when the degree of the punch d is equal to two or it goes to infinity respectively. This describes the problem of interaction between a biosample and a probe in the case that the probe shape near the tip has some deviation from its nominal shape and the shape function can be approximated by a monomial function of radius. We show that the generalized problems for monomial punches can be solved not only for frictionless but also for non-slipping boundary conditions. It is shown that regardless of the boundary conditions, the solution to the problems is reduced to the same dimensionless relations between the actual force, displacements and contact radius. Explicit expressions are derived for the values of the pull-off force and for the corresponding critical contact radius.

To consider the problems for cells we assume that first the cell membrane is prestressed and then it comes into contact with a probe or another cell; the stress field due to the contact is just a small perturbation of the large initial stresses; and the initial stress field can be considered as homogeneous. We show that the obtained results and in particular the JKR theory for a sphere adhesion can be extended to transversely isotropic [2] and prestressed materials [8]. We also show how the values of the effective contact modulus for two cells or between a cell and material of the probe and the work of adhesion for the same pairs may be quantified from a single test using a simple and robust BG method [9, 10]. The method is based on an inverse analysis of a stable region of the force–displacements curve obtained from the depth-sensing indentation of a sphere into an elastic sample. Of course, the results for the effective contact modulus depend on the employed model of non-linear elasticity. In particular, for the Mooney-Rivlin, harmonic, Treloar (neo-Hookean solid), and Bartenev-Khazanovich potentials the solutions are given explicitely. Note that the cell membranes are often described as materials of the neo-Hookean type.

The obtained results are related to (i) nanomanipulation of biological cells; (ii) probing of cell membranes by AFM; (iii) determination of the elastic modulus of biomaterials and biological samples; (iv) determination of the work of adhesion for two cells or a cell and an artificial material of a probe.

References

[1] Michael, K.E., García, A.J. Cell Adhesion Strengthening: Measurement and Analysis. In: ”Methods in Cell Biology, Vol. 83. Cell Mechanics, Eds. Wang, Y.-L. and Discher, D.E.”, Elsevier, Amsterdam, 330–347 (2007).

[2] Borodich, F.M., Galanov, B.A., Keer, L.M., Suarez-Alvarez, M.M., ”The JKR-type adhesive contact problems for transversely isotropic elastic solids”, Mechanics of Materials, 75, 34–44, (2014).

[3] Kendall, K., Kendall, M., Rehfeldt, F., ”Adhesion of Cells, Viruses and Nanoparticles”, Springer, Dordrecht (2011).

[4] Chu, Y.S., Dufour, S., Thiery, J.P., ”Johnson-Kendall-Roberts theory applied to living cells”, Phys. Rev. Lett., 94, 028102, (2005).

[5] Korayem, M.H., Rastegar, Z., Taheri, M., ”Application of Johnson-Kendall-Roberts model in nanomanipulation of biological cell: air and liquid environment”, Micro & Naono Letters, 7, 576–580, (2012).

[6] Wang, N., Tolić-Nørrelykke, I.M., Chen, J., Mijailovich, S.M., Butler, J.P., Fredberg, J.J. and Stamenović, D. ”Cell prestress. I. Stiffness and prestress are closely associated in adherent contractile cells”, American J. Physiology - Cell Physiology, 282, C606–C616, (2002).

[7] Borodich, F.M., Galanov, B.A., Suarez-Alvarez, M.M., ”The JKR-type adhesive contact problems for power-law shaped axisymmetric punches”, J. Mech. Phys. Solids, 68, 14–32, (2014).

[8] Borodich, F.M., ”The Hertz-type and adhesive contact problems for depth-sensing indentation”, Advances in Applied Mechanics, 47, 225–366, (2014).

[9] Borodich, F.M., Galanov, B.A., ”Non-direct estimations of adhesive and elastic properties of materials by depth-sensing indentation”, Proc. R. Soc. Ser. A, 464, 2759–2776 (2008).

[10] Borodich, F.M., B.A. Galanov, S.N. Gorb, M.Y. Prostov, Y.I. Prostov and M.M. Suarez- Alvarez, ”Evaluation of adhesive and elastic properties of polymers by the BG method”. Macromolecular Reaction Engineering, 7, 555-563, (2013).

Monday, March 30, 2015

Fourth Session

16:30-18:00

Tailoring the lubricity of mucin films at a hydrophobic inteface

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Keywords: mucin, mucoadhesive polymer, lubrication, denaturant, solvent quality

Mucins are a major macromolecular component of mucus gels that are known to protect underlying epithelial surfaces against pathogen and mechanical insult. Mucins and mucus gels are renowned for their unique lubricity, not only for biological tissues, but also for various engineering materials in aqueous environment [1,2]. While previous studies have mainly focused on elucidating the generic lubricating behavior of mucin itself, in this study, we present that the lubricity of mucin films can be tailored by a number of different approaches, including purification, denaturants, mucoadhesive polymers, and solvent quality control, at a compliant sliding interface of self-mated polydimethylsiloxane (PDMS).

Firstly, commercially available mucins are known to contain non-ignorable amount of impurities. Purification of BSM by anion exchange chromatography allowed us to discriminate the lubricating properties of BSM as influenced by impurities [3]. The presence of BSA was chiefly responsible for higher frictional forces observed from BSM samples as received from the manufacturer. The mechanisms contributing to higher friction forces by BSA were different at different contact length scales. At the macroscale contact, higher friction forces were caused by faster and dominant adsorption of BSA into the contacting area under a continuous cycle of desorption and re-adsorption of the macromolecules from tribostress. At the nanoscale contact, however, no significant desorption of the macromolecules is expected in tribological contacts because of too small contact area and extremely small number of BSM molecules involved in the contact area. Instead, increasingly higher friction forces with increasing amount of BSA in BSM layer are attributed to higher viscosity caused by BSA in the layer [4].

Secondly, porcine gastric mucin (PGM) is known to reveal poor lubricity at neutral pH [1]. Nevertheless, exposure of PGM in denaturant, such GuHCl 6 M + 10 mM DTT, prior to the dialysis against deionized water showed that the lubricating properties of PGM solution were significantly improved in neutral pH aqueous solution. Adsorbed mass onto PDMS surface was not virtually altered by the treatment with denaturant, but the adsorption strength appeared to be substantially improved by denaturation of C- and N-terminal regions and may account for the improved lubricity.

Thirdly, mucoadhesive polymers can be employed to enhance the lubricity of mucins layers. For example, a synergetic lubricating effect between porcine gastric mucin (PGM) and chitosan based on their mucoadhesive interaction was observed [5]. In acidic solution (pH 3.2) and low concentrations (0.1 mg mL^{-1}), the interaction of PGM with chitosan led to surface recharge and size shrinkage of their aggregates. This resulted in higher mass adsorption on the PDMS surface with an increasing weight ratio of [chitosan]/[PGM + chitosan] up to 0.50. While neither PGM nor chitosan exhibited slippery characteristics, the coefficient of friction being close to 1, their mixture improved considerably the lubricating efficiency (the coefficient of friction is 0.011 at an optimum mixing ratio) and wear resistance of the adsorbed layers. These findings are explained by the role of chitosan as a physical crosslinker within the adsorbed PGM layers, resulting in higher cohesion and lower interlayer chain interpenetration and bridging.

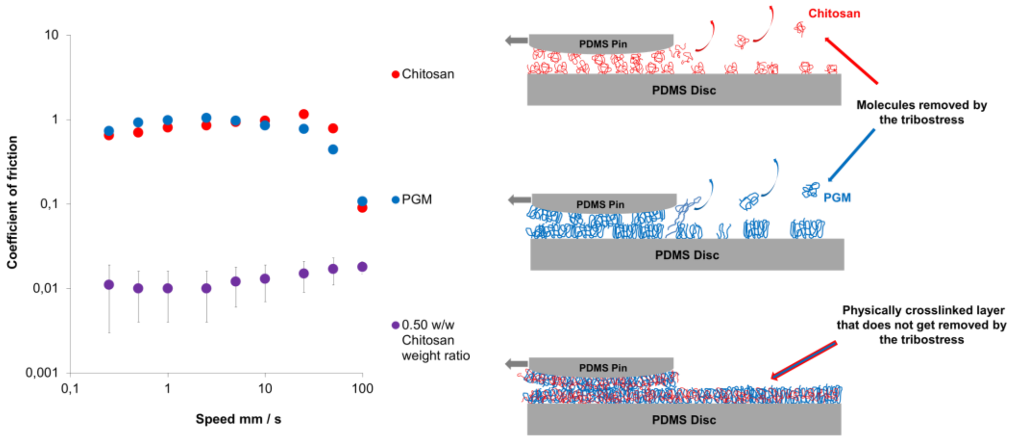

Figure 1: Illustration of the tribostress-induced removal of the adsorbed layers from the PDMS surface and their resistance to it in the presence of chitosan

Lastly, as with many other synthetic and biological macromolecules, the lubricating properties of mucins are significantly influenced by solvent quality. For example, by addition of methanol or ethanol into the aqueous mucin solutions, the lubricity can be either improved or degraded, depending on the concentration of alcohols and type of mucins. Changes in surface adsorption kinetics and conformation of mucins on PDMS surfaces as a result of degraded solvent quality are considered to be responsible for the altered lubricity.

Acknowledgements

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References

[1] Lee, S., Müller, M., Rezwan, K., Spencer, N.D., " Porcine gastric mucin (PGM) at the water/poly(dimethylsiloxane) (PDMS) interface: Influence of pH and ionic strength on the adsorption and aqueous lubrication properties", *Langmuir*, **21**, 8344-8353 (2005).
[2] Yakubov, G.E., McColl, J., Bongaerts, J.H.H., Ramsden, J.J., "Viscous boundary lubrication of hydrophobic surfaces by mucin", *Langmuir*, **25**, 2313-2321 (2009).
[3] Madsen, J.B., Pakkanen, K.I., Duelund, L., Svensson, B., Abou Hachem, M., Lee, S., "A Simplified Chromatographic Approach to Purify Commercially Available Bovine Submaxillary Mucins (BSM)", *Prep. Biochem. Biotech.*, **45**, 84-99 (2015).
[4] Nikogeorgos, N., Madsen, J.B., Lee, S., *Coll. Surf. B: Biointerfaces*, "Influence of impurities and contact scale on the lubricating properties of bovine submaxillary mucin (BSM) films on a hydrophobic surface", **122**, 760-766(2014).
[5] Nikogeorgos, N., Efler, P., Kayitmazer, A.B., Lee, S., ""Bio-glues" to Enhance Slipperiness of Mucins: Improved Lubricity and Wear Resistance of Porcine Gastric Mucin (PGM) Layers Assisted by Mucoadhesion with Chitosan", *Soft Matter* in press. DOI: http://dx.doi.org/10.1039/C4SM02021A (2015).

A new procedure for the solution of plane contacts

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Keywords: Plane contacts, Muskhelishvili potential, explicit equations.

Elastic plane contacts, although a simplification of the reality, even today offer a good and easy approximation for many practical/usual contacts in mechanical engineering. Despite of the simplifications made in these models, only a few and very simply contact pairs are analytically solved in the bibliography. Furthermore, the amount of these solutions decreases if the interior stress field is required, and only in very seldom cases explicit equations are presented.

Knowing this, the authors present a valuable procedure used to calculate the stress field in plane contacts between a punch and a half-plane. It is based on a certain property of Cauchy line integrals and a rewrite of the Flamant's equations for the half-plane. This method is applied in two steps. The first one greatly simplifies both the obtention of complex Muskhelishvili potential and, in the case in which friction and tangential load are present, the calculation of direct stresses produced on the contact surface [1]. The second step allows calculating directly and explicitly all of the components of the subsurface stress field once the surface contact stress distributions are known [2]. These explicit results are expressed in terms of the Muskhelishvili potential, which as indicated above, are easily calculated with the method proposed.

With the aid of these procedures, two previously unsolved problems, and relevant from the engineering point of view, were solved:

- The interior half-plane stress field generated in Carter's solution for an elastic rolling contact [2], [3].

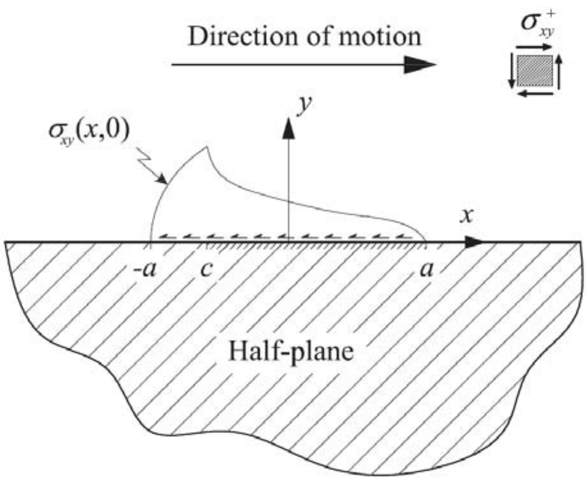

Figure 1: Surface shear stress generated in a rolling contact.

- The sub-surface half-plane stress field resulting from the contact between a half-plane and a flat rounded punch [4].

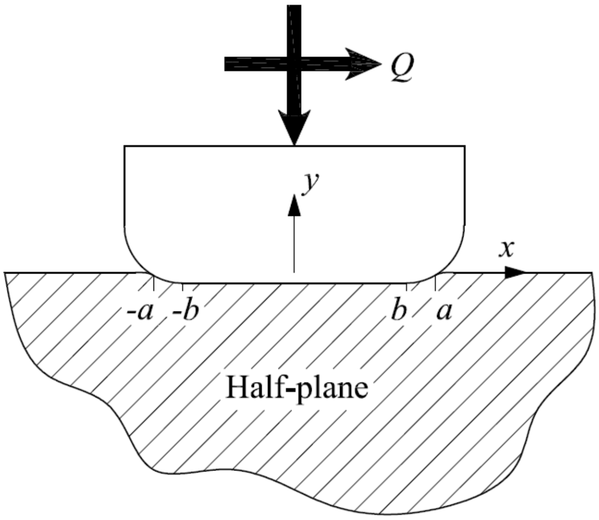

Figure 2: Contact pair between a flat rounded punch and a half-plane.

References

[1]. Vázquez, J., Navarro, C., Domínguez, J., "A new method for obtaining the stress field in plane contacts", International Journal of Solids and Structures, **49**, 3659-3665 (2012).

[2]. Vázquez, J., Navarro, C., Domínguez, J., "Explicit equations for sub-surface stress field in plane contacts", *International Journal of Mechanical Sciences*, **67**, 53-58 (2013).

[3]. Carter, F.W., "On the action of a locomotive driving wheel", *Proc. R. Soc. A*, **7**, 112-151 (1926).

[4]. Vázquez, J., Navarro, C., Domínguez, J., "Explicit equations for the half-plane sub-surface stress field under a flat rounded contact", *Journal of Strain Analysis for Engineering Design*, **49**, 562-570 (2014).

Characterization of Functional Surfaces of Advanced Materials and Coatings by Local Mechanical Contact Testing

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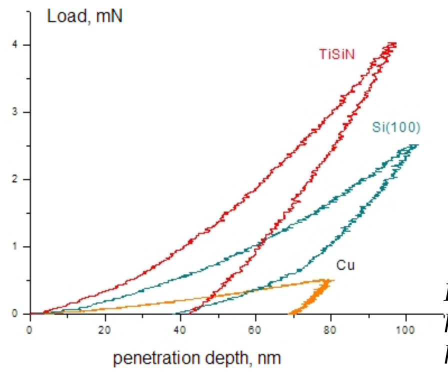
Keywords: Functional Surfaces, Nanoindentation, Wear resistance.

The experimental data obtained due to mechanical contact at instrumented indentation and sliding tests of functional surfaces of nanostructured coatings, quasicrystalline coatings and advanced composites are presented and analyzed.

Advanced thin solid films, multi-layered and graded materials are presently in the forefront of materials research and have a broad range of application because have possess the functionally modified surface, so called “functional surface”. Their near surface layer differs at nanometer scale from the rest part of bulk material in the properties due to proximity of interfaces and changes of chemical composition, type of structure, specific defects, etc.

The aim of this talk is to demonstrate that the judicious choice of a method of measurements and testing conditions is a key to reveal intrinsic properties of the object of study and also to discuss the received original experimental results. Some results obtained using modern methods for determining the hardness, Young’s modulus, adhesive/cohesive strength, friction coefficient, and wear resistance are shown below; they also published elsewhere [1-3].

As it is shown at Fg.1 indentation behavior of a number of advanced surfaces differs from the common ones, as their elastic recovery at unloading is higher by a factor of 2–3 as compared to common metallic alloys. It is true for both thin coatings and solid substrates [2].



Material	H, GPa	E, GPa	R, %
Copper	2.1	121	14
Titanium alloy (OT4-1)	4.1	130	19
Multi-layered Ti/α-C:H film	8.0	128	34
Metallic glass Zr-Cu-Ti-Ni (melt spun ribbon)	11.5	117	42
Silicon (100)	11.8	174	62
Ti-Si-N thin film	28.4	295	62

Elastic Recovery $R=(h_m-h_f)/h_m$,
h_m – maximum penetration depth,
h_f – residual (final) penetration depth

Figure 1: Curves of nanoindentation and calculated mechanical properties

Fine position control (250 nm and less), extremely low loads (0.5 mN and less) of modern testers gives a chance to penetrate solids with very delicate imprints (within 1 μm in lateral size) and characterize separately structure elements by instrumented indentation (ISO 14577). Two examples of imprints are presented at Fig.2: AFM image of polished cross-section amorphous metallic microwire covered by glass shell [3] and optical image of polished Al-based sintered composite added by Al-Cu-Fe quasicrystals [2]. The former nanoindentation study was done to estimate internal stresses at microwires using separate nanoindentation data on Young moduli for metallic core and glass shell. The latter nanoindentation was performed to explain an increase of wear resistance of Al-based sintered composite added by Al-Cu-Fe quasicrystals as shown Fig. 3.

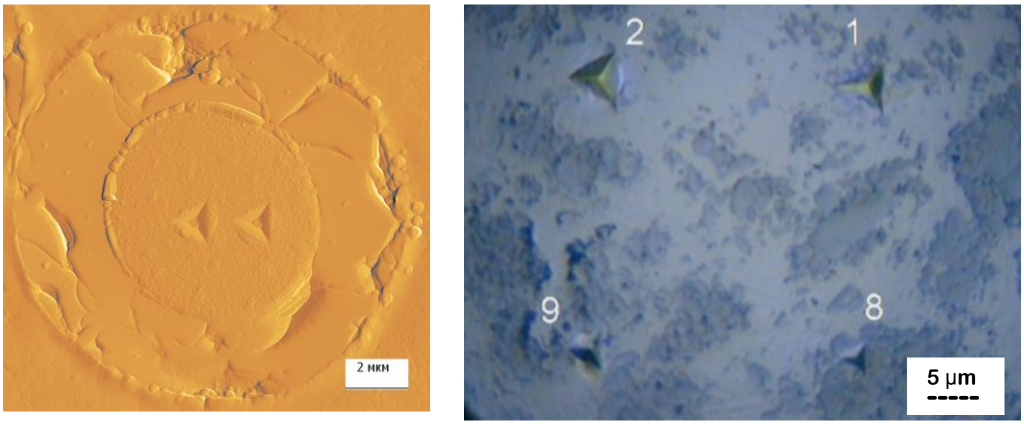


Figure 2: Imprints on glass covered amorphous microwire (left) and Al based sintered samples added with quasicrystalline Al-Cu-Fe particles (right): inpint 2, H=1,6 GPa, E=74 GPa, R=11,9 %; imprint 8, H=4,8 GPa, E=92 GPa, R=27,4 %.

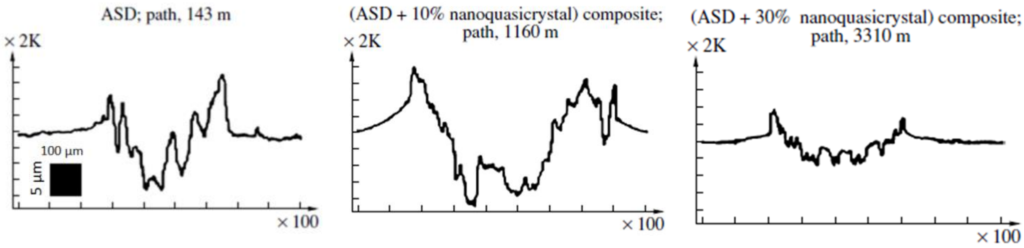


Figure 3: Wear track profiles of Al based sintered samples added with quasicrystalline particles

Acknowledgements

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References

[1] E.A. Levashov et al. Nanostructured titanium alloys and multicomponent bioactive films: Mechanical behavior at indentation, Materials Science & Engineering A, **570,** 51–62 (2013)
[2] M.I. Petrzhik, E.A. Levashov, Crystallography Reports, **52,** 966-974 (2007).
[3] E.V. Zamyatkina, M.I. Petrzhik. Russian Metallurgy (Metally) **4**, 376-378 (2011)

Tuesday, March 31, 2015

First Session

9:00-10:30

Determination of the effective interfacial conditions for Stokes flow on a rough solid surface

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The Stokes flow of a fluid in a channel bounded by two parallel solid walls is studied. The surface of a solid wall being assumed to be smooth, the classic perfect adherence condition is adopted for the corresponding homogeneous fluid-solid interface. The surface of the other wall being taken to be rough and capable of trapping small pockets of air, the corresponding liquid-solid interface is heterogeneous. This work aims to homogenize the heterogeneous liquid-solid interface so as to replace it by an imperfect homogeneous fluid-solid interface characterized by an effective slip length. A semi-analytical approach is developed for determining such an effective slip length when the rough surface is periodic but the distance between the two solid walls is arbitrary. The results obtained by the semi-analytical approach are compared with the relevant results delivered by the finite element method in a number of representative situations.

Stokes problem with threshold slip boundary conditions. Part I: theory (J. Haslinger); Part II: hydrophobic surfaces modelling (R. Kučera)

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Keywords: Stokes problem, slip boundary condition, adhesion force, active set algorithm, interior point method.

Observing a fluid flow along a solid impermeable wall, one can notice in some applications a variable tangential velocity of the fluid that may depend on a material quality or a shape of the wall. Such behaviour of the fluid is usually simulated by a slip type boundary condition (modelling the blood flow, metal forming processes, the polymer flow, or the hydrodynamics problems; see [1, 2] and references therein. Conditions of this type are used also in contact problems of solid mechanics, where they describe friction on common interfaces [3].

This contribution deals with the Stokes flow and threshold slip boundary conditions. A finite element approximation of the problem leads to the minimization of a non-differentiable energy functional J subject to two linear equality constraints: the impermeability condition on the slip part of the boundary and the incompressibility of the fluid. These conditions together with the non-differentiability of J will be released by Lagrange multipliers. Eliminating the velocity component, one gets the smooth dual functional in terms of three Lagrange multipliers (pressure in the domain,and shear and normal stress on the slip part).The solution to the dual problem is computed by an active set strategy [4] or by a path-following variant of the interior-point method [5]. The computational efficiency of the algorithms has been presented in [6].

Selected numerical experiments describing behaviour of fluids in contact with solid hydrophilic or hydrophobic surfaces [7] will be presented. Hydrophilicity is characterized by a capillary elevation in which liquid molecules are attracted to the surface so that the liquid adheres to the surface. The opposite case is a capillary depression when molecules are repelled from the surface that leads to hydrophobicity. In this case the liquid does not wet the surface and the slip depends on the shear stress. It is known from experiments that hydrophobic surfaces are also aerophilic, i.e. the ability to bind the air molecules to the surface. Therefore, for a certain kind of fluids, a thin layer of gas is formed between the fluid and the surface. The slip then occurs when a certain value of the shear stress is achieved. Navier [8] has already assumed that flow velocity and shear stress on the surface are related by

$$u_t = -k\sigma_t \,,$$

where k characterizes the effect of adhesion forces. This contribution deals with the threshold slip boundary which can be considered as a limit case of the Navier condition. One of our objectives is to analyze how the normal stress σ_N depends on k.

Acknowledgements

This work was supported by the European Development Fund in the IT4Innovations Centre of Excellence (CZ.1.05/1.1.00/02.0070) and by the project New creative teams in priorities of scientific research (CZ.1.07/2.3.00/30.0055) supported by Operational Programme Education for Competitiveness and co-financed by the European Social Fund and the state budget of the Czech Republic. The first and the second authors acknowledge the support of the Grant Agency of the Czech Republic (P201/12/0671). The third author also acknowledge the support of the Grant Agency of the Czech Republic (GA 13306001).

References

[1] Rao, I. J., Rajagopal, K. R., The effect of the slip boundary condition on the flow of fluids in a channel, Acta Mechanica, Vol. 135, pp. 113-126, (1999).

[2] Ayadi, M., Baffico, L., Gdoura, M. K., Sassi, T., Error estimates for Stokes problem with Tresca friction conditions, ESAIM: Mathematical Modelling and Numerical Analysis, (accepted 2014).

[3] Haslinger, J., Hlaváček, I., Nečas, J., Numerical methods for unilateral problems in solid mechanics, Analysis, Volume IV, Part 2, North Holland, Amsterdam, (1996).

[4] Dostál, Z., Optimal quadratic programming algorithms: with applications to variational inequalities, SOIA 23, Springer US, New York, (2009).

[5] Kučera, R., Machalová, J., Netuka, H., Ženšák, P., An interior point algorithm for the minimization arising from 3D contact problems with friction, Optimization Methods and Software, Vol. 28:6, pp. 1195-1217, (2013).

[6] Kučera, R., Haslinger, J., Šátek, V., Jarošová, M., Efficient methods for solving the Stokes problem with slip boundary conditions, submitted to Mathematics and Computers in Simulation, (2014).

[7] Brdička, M., Samek, L., Bruno, S., Mechanika kontinua, Academia, (2011).

[8] Navier, C.L.M.H., Mem.Acad.R.Sci.Inst.France I (1823) 414.

Tuesday, March 31, 2015

Second Session

11:00-13:00

A numerical study of the contact between rough surfaces: mechanical and transport phenomena at small scales

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Keywords: roughness, normal contact, sealing.

We study the frictionless and non-adhesive contact between elastic and elasto-plastic solids with randomly rough surfaces. Because of this roughness (inevitably present in most natural and engineering surfaces), when these solids are brought in contact, the ¡real contact area¿ - islands on which two solids touch each other - presents only a small portion of the nominal contact area. In the first part, we study (1) how this real contact area evolves with increasing pressure applied on the solids and (2) how this evolution and the morphology of contact can be linked with material and surface properties. In this context, we will discuss some details of generation of synthetic fractal surfaces, their characterization and the interplay of parameters: root mean squared height and slope, Nayak's parameter, Gaussianity, cutoff wavelengths and fractal dimension [6]. We will also briefly expose the numerical methods, that we use to carry out simulations of a mechanical boundary value problem with contact constraints: a finite element method [5] and an FFT boundary element method [4]. The numerical results will be compared with analytical models: asperity based models [1, 2] and Persson's contact theory [3].

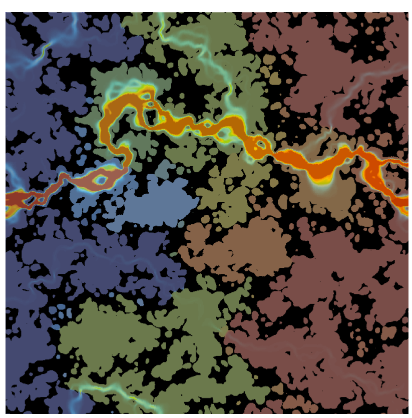

Figure 1: Near the percolation threshold, fluid flow through the free volume between rough surfaces in contact. Different colors correspond either to the flux intensity of hydrostatic fluid pressure; black zones are the contact regions

In the second part, we will present new results on simulation of transport phenomena weakly coupled with mechanical contact resolution: First, we will consider (3) leakage and percolation of a viscous fluid through the interface between rough surfaces (Fig. 1). Some generalization on the leakage regimes and the percolation limit will be suggested.

Acknowledgements
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References

[1] A. W. Bush, R. D. Gibson, and T. R. Thomas. The elastic contact of a rough surface. *Wear*, 35(1):87–111, 1975.

[2] G. Carbone and F. Bottiglione. Asperity contact theories: Do they predict linearity between contact area and load? *J Mech Phys Solids*, 56:2555–2572, 2008.

[3] B. N. J. Persson. Theory of rubber friction and contact mechanics. *J Chem Phys*, 115:3840–3861, 2001.

[4] H. M. Stanley and T. Kato. An FFT-based method for rough surface contact. *J Tribol-T ASME*, 119:481–485, 1997.

[5] V. A. Yastrebov. *Numerical Methods in Contact Mechanics*. ISTE/Wiley, 2013.

[6] Vladislav A. Yastrebov, Guillaume Anciaux, and Jean-François Molinari. From infinitesimal to full contact between rough surfaces: evolution of the contact area. *International Journal of Solids and Structures*, 52:83–102, 2015.

Analytical evaluation of the peak contact pressure in a rectangular elastomeric seal with rounded edges

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Keywords: Contact problem, analytical solution, elastomeric seal.

Rectangular seals constitute an alternative design to O-rings. Rectangular seals are employed in demanding applications such as aircraft actuators, e.g. ref. [1]. The seal edges are generally rounded, ref. [2]. As a consequence of the presence of filleted edges, the contact pressure exhibits Hertzian-type local bumps in its lateral zones; it remains almost flat in the central zone of the contact. The lateral bumps and the central flattish zone confer to the contact pressure distribution a camel-backed profile, see ref. [2], and ref. [3] for a similar axisymmetric problem.

It is difficult to derive a rigorous, analytical expression of the contact pressure curve for the title problem. In fact, the analytical solution available for a rectangular punch with rounded edges indenting a half plane, e.g. ref. [4] and related bibliography, is exact only in the situation of a rigid punch indenting a deformable half plane, ref. [5], whereas in the title problem the punch (i.e., the seal) is flexible and the half plane (i.e., the counterface) is rigid.

It has recently been shown in refs [5-7] that the unrealities of the above analytical solution may be corrected by combining the analytical solution with Fracture Mechanics (FM) results dealing with the stress singularities at the tip of a transverse crack in a strip of finite width. In this paper, an extension of formula (20) of ref. [5] is developed, that accounts for the combined effects of a) the presence of a filleted edge, and b) a finite seal width and height.

1. Purposely developed FM solution

The unrealities of the analytical solution of ref. [4] may be corrected by performing an asymptotic matching between two asymptotic solutions, namely a) the analytical pressure profile valid for a frictionless, plane, rounded indenter of semi-infinite width, and b) the corresponding pressure profile describing the sharp-edged equivalent of the problem under scrutiny.

The sharp edged model should account for two main peculiarities of the seal deformation. First, it should account for the effect of the seal material protruding from the contact region. Secondly, it should account for two parts of the rectangular seal border remaining rectilinear under compression, namely a) the flat portion of the sealing profile, and b) the seal opposite side. These two properties may be mimicked by a purposely developed FM solution referring to a cracked strip of finite width, and by imposing that the strip exhibits two transverse symmetry axes, one axis passing through two collinear edge cracks, and a second axis parallel to the previous one and at a distance equal to the seal height. A suitable FM model is therefore an infinitely long strip of finite width under an imposed elongation, exhibiting an infinite array of equispaced, transverse, collinear edge cracks. The above FM problem has not been treated in the pertinent literature, see ref. [8], p. 285 for a similar case. A FM solution has therefore been developed with the aid of FE.

Three singularities are considered, namely a) the presence of lateral collinear cracks, whose length is *a*; b) the presence of a central ligament, which may become vanishingly small, and whose half extent is denoted by *c*; c) the distance *2h* between two parallel, contiguous cracks. The width *w* of the cracked strip is therefore *2(a+c)*.

To derive a formula expressing the classical FM parameter K_I, following e.g. ref. [8], an equivalent length l_{eq} is introduced together with the three auxiliary variables α,β,γ:

$$\frac{1}{l_{eq}}=\frac{1}{a}+\frac{1}{c}+\frac{1}{h};\alpha=\frac{l_{eq}}{a};\beta=\frac{l_{eq}}{c};\gamma=\frac{l_{eq}}{h};K_I=\left(\mathrm{C}_1+\mathrm{C}_2\ \alpha+\mathrm{C}_3\ \beta+\mathrm{C}_4\ \gamma\right)\frac{\delta}{h}E'\sqrt{\pi l_{eq}} \qquad (1)$$

where δ is the elongation and *E'* is an equivalent elastic modulus, and the numerical coefficients of the bracketed polynomial have been calibrated with a Finite Element (FE) analysis.

2. Asymptotic matching

By performing an asymptotic matching, the following formula for the maximum contact pressure is obtained:

$$p_{\max}\cong C\sqrt[3]{\frac{9\pi K_I^2 E}{6r}} \qquad (2)$$

where *r* is the radius of the seal rounded edges, and *C* is a numerical constant. Since it is too difficult to derive analytically the applicability range of the previous formula, an error analysis is under development versus FE forecasts, involving more than one thousand geometries; preliminary, promising results are presented, that indicate a maximum error of about 20 per cent.

References

[1] Ruskell, L.E.C., "A rapidly converging theoretical solution of the elastohydrodynamic problem for rectangular rubber seals", *Journal of Mech. Engng Science*, **22**, 1, 9-16 (1980).

[2] Strozzi, A.,"Static stresses in an unpressurized, rounded, rectangular, elastomeric seal", *ASLE transactions*, **29**, 4, 558-564 (1986).

[3] Strozzi, A., Baldini, A., Giacopini, M., Bertocchi, E., Bertocchi, L., "Normalization of the stress concentrations at the rounded edges of a shaft–hub interference fit", *Journal of Strain Analysis for Engineering Design*, 0309324711403845 (2011).

[4] Ciavarella, M., Hills D.A., Monno, G., "The influence of rounded edges on indentation by a flat punch", *Proc. I. Mech. E, Part C: Journal of Mech. Engng Science*, **212**, 212-319 (1998).

[5] Strozzi, A., Bertocchi, E., Baldini, A., Giacopini, M., "On the applicability of the Boussinesq influence function in modelling the frictionless elastic contact between a rectangular indenter with rounded edges and a half-plane", *Proc. I. Mech. E, Part C: Journal of Mech. Engng Science*, 0954406214542641 (2014).

[6] Sackfield, A., Mugadu, A., Barber J.R., Hills D.A., "The application of asymptotic solutions to characterising the process zone in almost complete frictionless contacts", *Journal of the Mechanics and Physics of Solids*, **51**, 1333-1346 (2003).

[7] Banerjee, N., Hills, D.A., Dini, D., "The derivation and application of a semi-infinite flat and rounded asymptotic frictionless contact", *Int. Journal of Mech. Sciences*, **51**, 662-666 (2009).

[8] Tada, H., Paris P.C., Irwin, G.R., "The stress analysis of cracks handbook ASME", N.Y., 55-56 (2000).

A Combined Finite Element Framework for Contact and Fluid-Structure Interaction

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Keywords: Mortar contact, XFEM, FSI, cut library, ghost penalty stabilization, time integration.

Within this work, we develop a combined contact and fluid-structure interaction (FSI) framework based on a monolithic coupling scheme. The aim is to achieve a smooth transition between FSI and frictional dry contact. One of the major applications in biomechanics is the simulation of heart valves in a beating heart. Another example is the simulation of tires on a wet road in the field of automotive engineering.

For the contact discretization we use mortar finite element methods with finite deformations [1].

For FSI we employ a fixed-grid approach based on the extended finite element method (XFEM) [2]. Any moving-grid approach, e.g. arbitrary Lagrangean-Eulerian (ALE), would encounter serious problems in the case of approaching bodies, since fluid elements between contact interfaces would be completely squeezed together. While problems due to excessive mesh distortion could still be resolved by remeshing, the fluid domain eventually undergoes a topological change in the limit case of contact, which can only be accurately captured by a fixed-grid approach.

Previous work on the topic of FSI with contact has been done using a simple combination of these two methods, the mortar contact discretization and the XFEM FSI algorithm [3]. While many aspects of the resulting coupled FSI and contact formulation were already captured very well, this approach still leaves room for improvements. Three main challenges have been identified and solutions will be presented in this contribution:

1. The geometrical and algorithmic handling of overlapping cut meshes in the contact case
2. The numerical stabilization of the interface and the fluid when gaps become very small
3. The use of an appropriate time integration scheme for the transition between FSI and contact

In the context of the XFEM, a cut library is the algorithmic basis for handling and exactly representing complex geometrical scenarios in cut finite elements. This cut library provides a geometrical library for boolean operations on tetrahedral and hexahedral meshes as well as methods and data structures to facilitate the numerical integration of cut finite elements and moving interfaces. As the weak constraint enforcement by the mortar method allows for small local penetrations, the surfaces of structural bodies in contact may encounter small local overlaps. A key component enabling our XFEM FSI framework to treat the contact case correctly is to expand the cut library to handle overlapping structural surfaces (selfcut). The selfcut library replaces the surfaces of an arbitrary number of structural bodies by one new consistent surface mesh. This new mesh constitutes a suitable geometrical object for cutting the background fluid elements via our cut library.

To stabilize the interface and the fluid solution next to structural surfaces, face-oriented stabilization (FOS) is commonly used, which results in additional ghost penalty (GP) terms [2]. In case of nearly contacting bodies, the physical fluid domain will sooner or later reduce to less than one fluid element across the height of the fluid gap. Using first-order finite elements in the fluid gap results in a conflict between the weak enforcement of the no-slip boundary condition and the weak enforcement of the conservation of mass. To overcome this problem, we employ second-order finite elements for the fluid discretization. Moreover, we show that for higher-order finite elements it is advisable to express the interface and the fluid stabilization with the projection based GP operators.

An challenging task in fixed-grid moving boundary simulations is the development of a reasonable time integration scheme, since the geometry of the underlying fluid domain varies permanently over time. This issue is further amplified in case of topological changes such as two interfaces getting into contact or losing contact. Some approaches to tackle this problem are proposed in literature (e.g. [4]) and discussed in this talk.

In summeray, the main result of our work is the first finite element framework, which resolves the three major challenges of FSI simulations with contact. First, our approach enables an algorithmic handling of contact in an FSI simulation. Second, it incorporates a meaningful and stable fluid solution in small fluid gaps. And finally, it guarantees a smooth transition from FSI to frictional dry contact and vice versa.

An example for a 3D fluid simulation with several contacting bodies is shown in Figure 1.

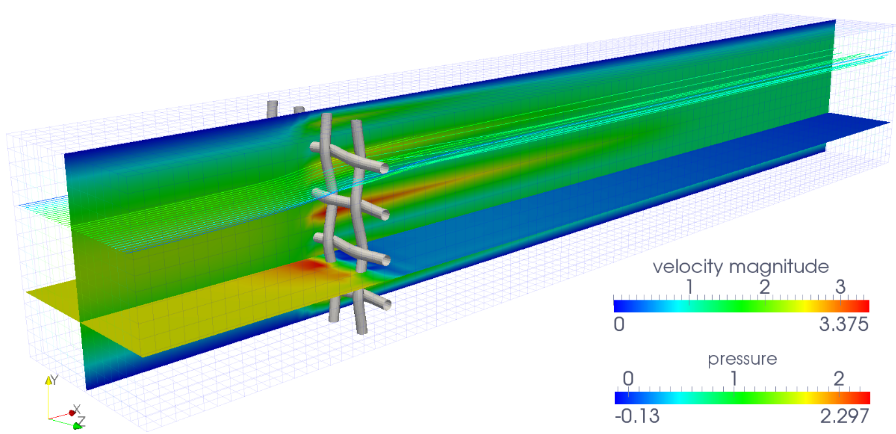

Figure 1: A grid of contacting structures in a fluid channel.

References

[1] Popp, A., Gitterle, M., Gee, M.W., Wall, W.A., "A dual mortar approach for 3D finite deformation contact with consistent linearization", Int. J. Numer. Meth. Engng, 83 1428-1465 (2010).

[2] Schott, B., Wall, W.A., "A new face-oriented stabilized XFEM approach for 2D and 3D incompressible Navier-Stokes equations", Comput. Methods Appl. Mech. Engrg., 276 233-265 (2014).

[3] Mayer, U.M., Popp, A., Gerstenberger, A., Wall, W.A., "3D fluid-structure-contact interaction based on a combined XFEM FSI and dual mortar contact approach", Comput Mech, 46 53-67 (2010).

[4] Henke, F., Winklmaier, M., Gravemeier, V., Wall, W.A., "A semi-Lagrangean time-integration approach for extended finite element methods", Int. J. Numer. Meth. Engng, 98 174-202 (2014).

Computational Contact Mechanics Methods for the Finite Cell Method

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Keywords: FCM, contact cell, cell surface, discrete cell.

1 Short Information about the Finite Element Cell Method

The motivation of the current work is to develop a set computational contact mechanics method which will be effective for the Finite Element Cell Method.

The Finite Cell Method (FCM) provides a method for the computation of structures which can be described as a mixture of high-order FEM and special integration technique. The method is one of the novel computational methods and is highly developed in numerous works of Düster and colleagues, see in [1]. Rough idea of FCM is illustrated in Fig. 1. Computationally the necessary area Ω is represented as $\Omega_e \supseteq \Omega$, whereas Ω_e is represented by simple geometry (rectangular element of high order). In the area Ω_e the indicator function α is introduced: $\alpha(x) = \begin{cases} 1, & x \in \Omega, \\ 0, & x \in \Omega_e \setminus \Omega. \end{cases}$.

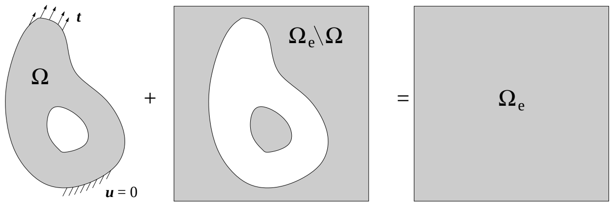

Figure 1: Extension of the physical area Ω to Ω_e [2]

Numerically, FCM is realized as integration by sub-domains, or by cells.One of the problem of FCM is also description of boundary conditions, see [2]. Advantages of FCM are represented in each problem where to hard FEM mesh:

- CT scan of human bodies;
- Modeling of the roughness of the surface;
- Modeling of 3D printing;
- etc etc etc

2 Development of the efficient computational contact method

One of the open problem is description the contact. There is almost straightforward implication of the contact for the high order method, see in [3], [4]. These methods are went later into iso-geometric method. However, the problem, namely for FCM is still remain open. The current development is mostly shown in 2D, for which the classical case FEM contact is shown in [5].

For FCM method we are developing and testing on the Hertz problem the following algorithms:

- CSTAS contact element (Cell-Surface-To- Analytical-Surface) for contact with rigid bodies;
- DCTC contact element (Discrete-Cell- To-Cell) – based on representation of the integration point as an discrete finite element for both deformable bodies;
- CSTCS contact element (Cell Surface to Cell Surface) – based on building the contact element between nearest integration points. It is appeared most robust element for deformable bodies, however with most complex matrix projections.

It is investigated optimal number of cells and integration points as well as the selection of the class of approximation functions and spline smoothing. All examples are tested for convergence, numerical error and special effects (smoothness). The most attractive example is the possibility to describe the self contact within only ONE ELEMENT.

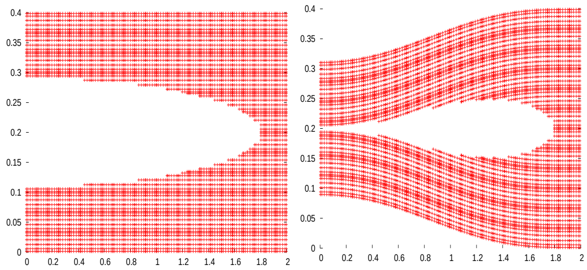

Figure 2: Self-contact in one element. 24 cells and 6 order of Bernstein polynomial.

There many points to discuss here: only non-penetration, or correctness for the contact stresses. All these depend on the selected method.

References

[1] A. Düster, J. Parzivian, Z. Yang, E. Rank: *The finite cell method for three-dimensional problems of solid mechanics,* Comput. Methods Appl. Mech. Engrg. 197 (2008) 3768-3782.

[2] J. Parzivian, A. D"uster, E. Rank: *Finite Cell Method: h- and p-extension for embedded domain problems in solid mechanics,* Computational Mechanics 41:121-133, 2007.

[3] A. Konyukhov, K. Schweizerhof: *Incorporation of contact for higher-order finite elements in covariant form,* Comput. Methods Appl. Mech. Engrg. 198 (2009) 1213-1223.

[4] D. Franke, A. Düster, V. N"ubel, E. Rank: *A comparison of the h-, p-, hp-, and rp-version of the FEM for the solution of the 2D Hertzian contact problem,* Comput. Mech. 45:513-522, 2010.

[5] A. Konyukhov, K. Schweizerhof *A special focus on 2D formulations for contact problems using a covariant description,* Int. J. Num. Mech. Engr 66 (9), 1432–1465, 2006.

Wednesday, April 1, 2015

First Session

09:00-10:30

A Framework for Quantification of Fretting fatigue

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The strength of notched components is well known to be adequately described by Williams' semi-infinite wedge solution; the local spatial distribution of stresses is properly represented and the multiplier on the solution, the generalised stress intensity factor, controls the magnitude, and hence the strength, at any rate for brittle materials. This has been shown by many people over a number of years. It is first shown, here, that the same solutions can be applied to quantify the fretting fatigue strength of sharp-edged contacts. Thus, provided that the coefficient of friction is sufficiently large for any slip zone present to be confined to a near-edge region, a wedge whose internal angle is the sum of the half-plane and the contact edge angle will adequately describe the state of stress at the edge of the contact, and hence the fatigue strength.

The next question is whether it is possible to devise a further asymptotic form applicable to incomplete contacts, and the answer is 'yes'. The local contact pressure here decays towards the contact edge in a square root bounded manner. And, if the friction is sufficiently high, the shear traction will be square root singular, and this field may be excited by either the application of shear or remote tension. In practice the singularity is relieved by the presence of a small slip zone, and the size of this is derived. We then go on to analyse a wide range of data from the open literature relating to fretting fatigue strength of incomplete contacts and show how this may be very satisfactorily collapsed onto a single curve for any given material. Size effects are also properly taken into account by this approach.

These two forms of correlation – one for incomplete contacts and one for complete contacts, enable the majority of fretting contact strengths to be measured in a simple laboratory experiment and then applied to a complicated prototype with confidence – precisely because the local fields are matched. The question remains whether these ideas may be extended to other types of contact – in particular, receding contacts - and whether a practically useful form may be evolved for this case. This is discussed, as is the value of frictional shakedown calculations in establishing whether these problems exhibit, in the steady state, cyclic slip. Many complete contacts would seem to become adhered (if not adhered from the outset) and so ultimately behave more like 'notch' problems.

On Gross Slip Wear of an Axisymmetric Punch Using MDR

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Keywords: Wear, gross-slip, method of dimensionality reduction

In the present paper, we study the development of wear profile under conditions of gross slip and assumption of the Reye-Archard wear criterion. Simulations are carried out using the Method of Dimensionality Reduction (MDR) and a full FEM formulation. The calculation time of MDR model is several orders lower than that of FEM-based models and allows for much higher spatial resolution.

In the most practical cases, for theoretical description of wear, the simplest wear equation is used stating that the wear volume is proportional to the dissipated energy and inversely proportional to the hardness σ_0 of the worn material (Reye, 1860, Khrushchov and Babichev, 1960, Archard and Hirst, 1956). The local formulation of this criterion means that the linear change $\Delta f(r)$ of the three-dimensional profile $f(r)$ (where r is the polar radius in the contact plane) is given by the equation

$$\Delta f(r) = \frac{k_{wear}}{\sigma_0}\tau(r)\left(\Delta u_x^{(0)} - \Delta u_x^{(3D)}(r)\right),$$

where $u_x^{(0)}$ is relative tangential displacement, $u_x^{(3D)}(r)$ is a part of relative tangential displacement due to elastic deformation of a medium, $\tau(r)$ is tangential stress and symbol Δ means the increment of corresponding parameter during one step of spatial displacement. In this wear law, all the influencing parameters are “absorbed” in the empirical dimensionless wear coefficient k_{wear}.

In the present paper we provide a solution with the so-called Method of Dimensionality Reduction, that recently has been applied for a simulation of fretting wear (Dimaki et al, 2014, Li et al, 2014, Popov, 2014, 2014a, 2014b) The results will then be compared with a full FEM simulation. In Figure 1a, the worn profiles (calculated both using the MDR (solid lines) and FE (dashed lines) are shown for different sliding length U related to the reference length U_0 that is necessary to obtain a wear depth comparable with the initial indentation depth d_0, $U_0 = \frac{\pi\sigma_0 a_0^2 d_0}{k_{wear}\mu F_N}$, where a_0 is the initial contacts radius. Pressure distribution is shown in Figure 2 (both MDR and FE simulations).

Parameter studies have shown that the calculated profiles slightly change with the number of elements used, and the convergence is achieved at approximately $N = 800$ elements. At that number of elements, the simulation up to "almost complete wear" takes about 1 minute for number of elements $N = 800$, while for $N = 500$ the results remain quite precise and the calculation takes about 10 second.

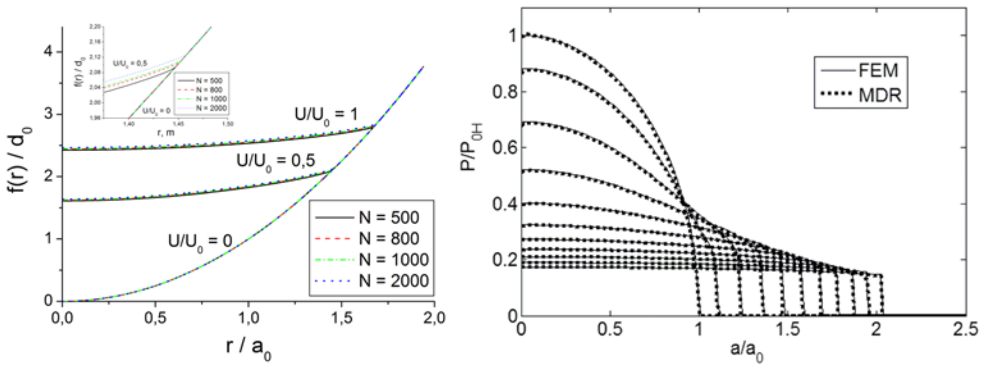

Figure 1: (a) Worn profiles and (b) Normal pressure distributions obtained in the presented MDR-based model and in three-dimensional FEM model.

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Acknowledgements

This work was partially supported by Deutsche Forschungsgemeinschaft (DFG), the Ministry of Education of the Russian Federation, and Deutscher Akademischer Austausch Dienst (DAAD).

References

[1] Archard, J.F., Hirst, W., 1956. The Wear of Metals under Unlubricated Conditions. *Proc. R. Soc. Lond. A*. **236**, 397-410, doi: 10.1098/rspa.1956.0144.

[2] Dimaki, A.V., Dmitriev, A.I., Chai, Y.S., Popov, V.L., 2014. Rapid simulation procedure for fretting wear on the basis of the method of dimensionality reduction. *Int. J. Solids Struct*., **51**, 4215-4220.

[3] Khrushchov, M.M., Babichev, M.A., 1960. Investigation of wear of metals (Исследование изнашивания металлов), Moscow, Russian Academy of Sciences.

[4] Li, Q., Filippov, A.E., Dimaki, A.V., Chai, Y.S., Popov, V.L., 2014. Simplified Simulation of Fretting Wear Using the Method of Dimensionality Reduction. *Phys. Mesomech.*, **17** (3), 236-241.

[5] Popov, V.L., 2014. Analytic solution for the limiting shape of profiles due to fretting wear, *Sci. Rep.* **4**, 3749.

[6] Popov V.L., Hess M., 2014a. Method of dimensionality reduction in contact mechanics and friction: a users handbook. I. Axially-symmetric contacts. *Facta Universitatis, Mech. Engng.*, **12** (1), 1-14.

[7] Popov, V.L., Heß, M., 2014b. Method of Dimensionality Reduction in Contact Mechanics and Friction, Springer.

[8] Reye, T., 1860. Zur Theorie der Zapfenreibung. *Der Civilingenieur*, **4**, 235-255.

An implicit finite wear contact formulation based on dual mortar methods

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Keywords: Mortar Finite Element Methods, Finite Deformation Contact, Wear, Shape Evolution, Arbitrary-Lagrangian-Eulerian formulation.

Wear phenomena are of great importance for industrial applications due to their destructive impact on machines and strongly loaded components. It is a process of material removal associated with frictional contact, which results in finite shape changes due to the accumulation of wear. Despite the existence of various different types of wear effects like abrasive, adhesive, corrosive and fretting wear, their problem kinetics are most commonly described by the phenomenological Archard's law [1]. It relates the amount of worn volume with the contact normal force, sliding length and a problem-specific wear coefficient.

In the past years, computational treatment of these phenomena was predominantly done by employing traditional node-to-segment contact formulations [2]. Nevertheless, mortar finite element methods have become the commonly accepted state-of-the-art approach for computational contact mechanics [3, 4] and are utilized as contact framework for this contribution. Here, Archard's wear law is reformulated to express the worn volume as interface displacements in negative normal direction and is weakly imposed following the segment-to-segment idea. The non-penetration and frictional sliding constraints are formulated as nonlinear complementarity problem, now also including consistent linearizations with respect to the additional nodal wear.

Constraint enforcement is realized via the Lagrange multiplier method employing so-called dual shape functions [5], which allow for static condensation procedures for the discrete Lagrange multipliers. Dual shape functions are now also used as testing functions for the weak enforcement of Archard's wear law, which opens up the possibility of applying an additional condensation procedure for the unknown nodal wear variables. This avoids any increase in the global system size of the linearized system of equations to be solved in each Newton-Raphson iteration step, while the full Lagrange multiplier information as well as the wear effects are included in the condensed system.

In addition, the underlying shape evolution problem is solved with the help of an Arbitrary Lagrangian-Eulerian (ALE) formulation, which guarantees proper mesh quality for the worn configuration. The shape evolution problem within the ALE step and the wear calculation within the classical structural step including contact are solved iteratively until convergence is achieved (partitioned solution scheme).

The developed algorithm is applicable to one-sided as well as two-sided wear for two- and three-dimensional problems, see Figure 1. Furthermore, treatment of quasi-steady-state wear phenomena and employing higher-order elements is possible and will be demonstrated in this contribution.

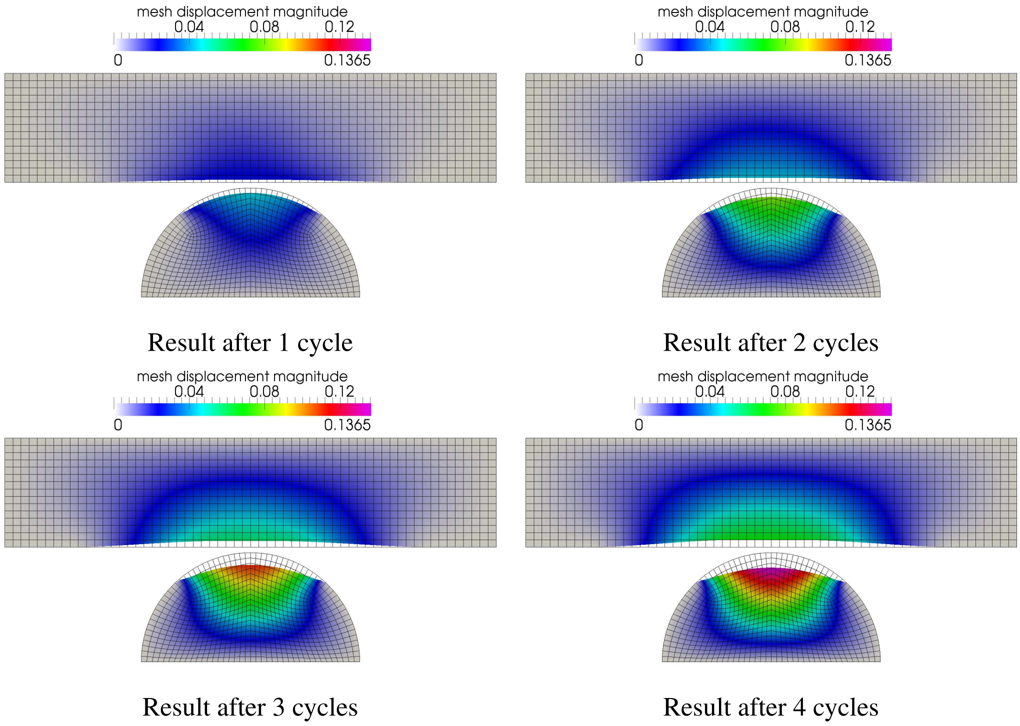

Figure 1: Visualized worn reference configuration with mesh displacements for oscillating beam with two-sided wear. Wire frame represents initial configuration.

References

[1] Archard, J. F., ”Contact and rubbing of flat surfaces”, Journal of Applied Physics, 24, 981–988 (1953).

[2] Lengiewicz, J. and Stupkiewicz, S., ”Continuum framework for finite element modelling for finite wear”, Computer Methods in Applied Mechanics and Engineering, 205-208, 178-188 (2012).

[3] Popp, A. and Gee, M. W. and Wall, W. A., ”A finite deformation mortar contact formulation using a primal-dual active set strategy”, International Journal for Numerical Methods in Engineering, 79, 1354–1391 (2009).

[4] Popp, A. and Gitterle, M. and Gee, M. W. and Wall, W. A., ”A dual mortar approach for 3D finite deformation contact with consistent linearization”, International Journal for Numerical Methods in Engineering, 83, 1428–1465 (2010).

[5] Wohlmuth, B. I., ”A mortar finite element method using dual spaces for the Lagrange multiplier”, SIAM Journal on Numerical Analysis, 38, 989–1012 (2000).

Wednesday, April 1, 2015

Second Session

11:00-13:00

A single state variable model for adhesive interfaces

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Keywords: Adhesive contact, cohesive interfaces, adhesive damage.

The object of this communication is a relatively simple and quite general framework for the description of the response of an adhesive surface separating two rigid bodies. This framework, proposed in [1] and originating from the "RCCM model" developed in [2] and [3], is based on the definition of a small number of fundamental objects:

- general laws, typically, energy conservation and dissipation principle, that is, mechanical versions of the first two laws of thermodynamics,
- a state variable α,
- an elastic potential and a set of dissipation potentials, in terms of which the general laws take specific forms,
- a set of constitutive assumptions.

These objects determine the evolution law for the state variable, and this law, in turn, determines the response to any given deformation process. A peculiarity of the model is that a single state variable keeps track of many dissipation sources, such as damage, viscosity, and friction. In the assumed constitutive laws, the intensity of adhesion is supposed to decrease under the action of prescribed tangential and normal relative displacements. This reduction is attributed to progressive damage. The assumed contact law is unilateral of the Signorini type for the normal displacements, and frictional of the Coulomb type for the tangential displacements.

The model's description starts with the simple case of a purely normal adhesive damage shown in Fig. 1a, where u denotes the normal relative displacement. The figure also shows that, for every given process $t \mapsto u_t$, the current value of the state variable is

$$\alpha_t = \max_{\tau \in [0,t]} u_\tau \,. \tag{1}$$

The constitutive assumption consists in prescribing the shape of the response curve $\sigma = f(u)$ at the first loading, and in assuming a linear unloading path down to the origin, with reloading along the

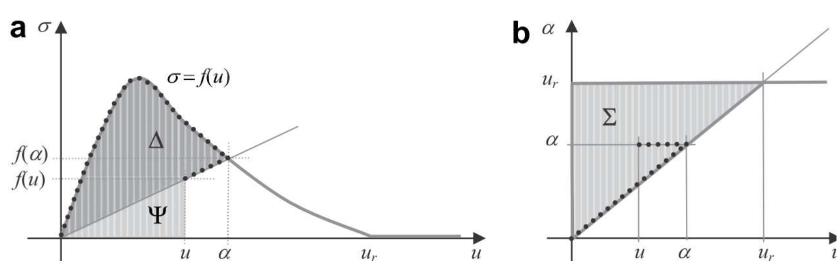

Figure 1: Adhesion with damage under normal loading. Response to a process of loading-unloading, represented in the force-displacement plane (a), and in the state space (b).

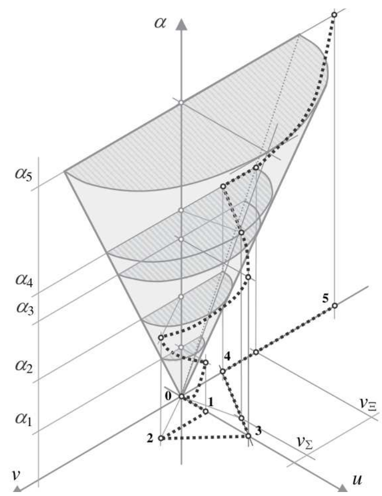

Figure 2: Adhesion with damage, viscosity, and friction. Response to a process $t \mapsto (u_t, v_t)$, represented in the state space (u, v, α).

same path. The areas marked by Ψ and Δ measure the elastic energy and the dissipation associated with the current state (u_t, α_t). The energy conservation leads to the power equation $P = \dot{\Psi} + \dot{\Delta}$, where $P = \sigma \dot{u}$ is the power supplied by the exterior. This equation has the explicit form

$$\sigma \dot{u} = \frac{f(\alpha)}{\alpha}\, u\dot{u} + \Big(\frac{f(\alpha)}{2\alpha}\Big)'(u^2 - \alpha^2)\,\dot{\alpha}\,, \tag{2}$$

with $()^{\cdot} = d/dt$ and $()' = d/d\alpha$. The response to a process of loading-unloading starting from the origin is shown in Figure 1a, while Figure 1b shows the same response represented in the state space (u, α). When including the tangential displacements v and the dissipation due to viscosity and friction, the power equation takes the form

$$\begin{aligned} \sigma\dot{u} + \tau\dot{v} = {} & \frac{f_N(\alpha)}{\alpha}\, u\dot{u} + \frac{f_T(\alpha)}{\alpha}\, v\dot{v} \\ & + \Big(\frac{f_N(\alpha)}{2\alpha}\Big)'(u^2 - \alpha^2)\,\dot{\alpha} + \Big(\frac{f_T(\alpha)}{2\alpha}\Big)'(v^2 - \alpha^2)\,\dot{\alpha} + \frac{1}{2}\, h(\alpha)\,\dot{\alpha}^2 + \mu(\alpha)\,\sigma^-|\dot{v}|\,, \end{aligned} \tag{3}$$

with f_N and f_T normal and tangential constitutive curves, and with h and μ material functions in the dissipative potential for viscosity and friction, respectively. Figure 2 shows the response to a process $t \mapsto (u_t, v_t)$ in the three-dimensional state space (u, v, α). It is clear that the model can be futher extended by adding other dissipation sources, each one represented by a new dissipation potential, and by including supplementary deformation parameters and state variables.

References

[1] Del Piero, G., Raous, M., “A unified model for adhesive interfaces with damage, viscosity, and friction”, Eur. J. of Mechanics A/Solids 29, 496-507 (2010).

[2] Raous, M., Cangémi L., Cocou M., “A consistent model coupling adhesion, friction and unilateral contact”, Comput. Methods in Appl. Mech. Engng. 177, 383-399 (1999).

[3] Raous, M., Monerie, Y., “Unilateral contact, friction and adhesion in composite materials”, in: Martins, J.A.C., Monteiro Marques, M. (Eds), “Contact Mechanics”, Kluwer, Dordrecht, 333-346 (2002).

Electro-mechanical coupling in cracked Silicon solar cells embedded in photovoltaic modules: experiments and simulations

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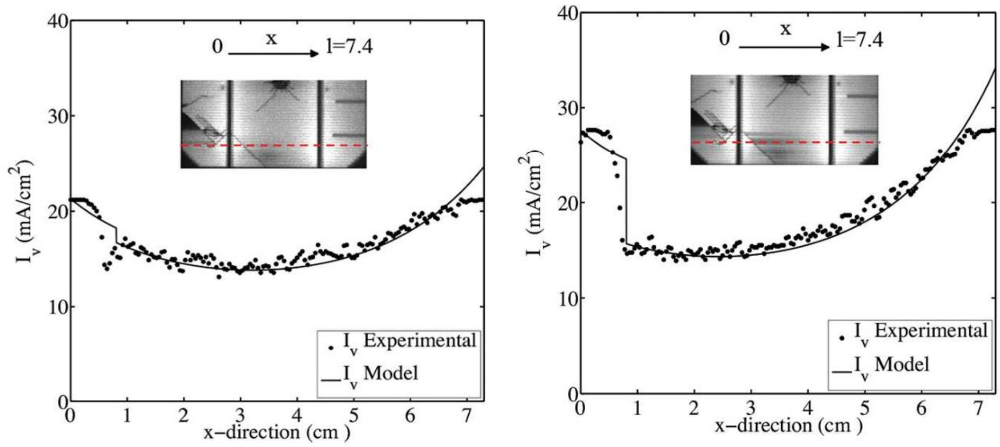
Keywords: Cracks, Silicon, electro-mechanical coupling.

Cracking in Silicon solar cells is an important factor for the electrical power-loss of photovoltaic modules. Simple geometrical criteria identifying the amount of inactive cell areas depending on the position of cracks with respect to the main electric conductors have been proposed in the literature to predict worst case scenarios [1].

Here we present an experimental and numerical study based on the electroluminescence (EL) technique showing that crack propagation in monocrystalline Silicon cells embedded in photovoltaic (PV) modules is a much more complex phenomenon. In spite of the very brittle nature of Silicon, due to the action of the encapsulating polymer and residual thermo-elastic stresses, cracked regions can recover the electric conductivity during mechanical unloading due to crack closure [2]. During cyclic bending, fatigue degradation is also reported. This pinpoints the importance of reducing cyclic stresses caused by vibrations due to transportation and use, in order to limit the effect of cracking in Silicon cells. An electric model with a localized electric resistance dependent on the crack opening is finally proposed to predict the electric response of electric conductors deposited on Silicon and intersected by a crack, see some results in Fig. 1 for two different mid-span deflection of a PV module tested in bending.



(a) 6 cm mid-span deflection

(b) 15 cm mid-span deflection

Figure 1: Electric current perpendicular to the solar cell along a conductor intersected by a crack, for different deflections of the photovoltaic module tested in bending.

Acknowledgements

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References

[1] Köntges, M., Kunze, I., Kajari-Schröder, S., Breitenmoser, X., Bjørneklett, B. " The risk of power loss in crystalline silicon based photovoltaic modules due to micro-cracks", *Solar Energy Materials and Solar Cells*, **95**, 1131–1137, (2011).

[2] Paggi, M., Berardone, I., Infuso, A., Corrado, M. "Fatigue degradation and electric recovery in Silicon solar cells embedded in photovoltaic modules", *Scientific Reports*, **4**, 4506, (2014).

On the problem of a Timoshenko beam bonded to an elastic half-plane

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Keywords: Timoshenko beam, half plane, Green function, Jacobi polynomials, collocation technique.

The contact problem of beams, rods, ribs and plates bonded to a half-plane has been widely investigated by many Authors. In particular, the problem of prismatic beams resting on a finite or semi-infinite elastic substrate deserves great interest because its practical applications in many engineering application. As an example, Shield and Kim (1992) investigated the problem of an Eulero-Bernoulli beam resting on an elastic half-plane under symmetric loading conditions, founding the interfacial stresses as well as the SIFs at the edges of the beam. The Authors also studied the effect induced by an elastic-perfectly plastic cohesive interface. Nonetheless, a complete analytical study of the contact problem of a Timoshenko beam bonded to a half-plane cannot be found in literature.

The present study concerns the contact problem of a Timoshenko beam resting on an elastic half-plane under general edge loading. Let N_0, T_0, M_0 external horizontal and vertical forces and bending moment, respectively, acting at the edge of a Timoshenko beam. Reference is made to a Cartesian coordinate system centered at the middle of the beam, as depicted in Figure 1. In absence of further external load distributions acting on the system, the equilibrium equations of the beam read:

$$N' + \tau = 0; \qquad T' + q = 0; \qquad M' - T' + \tau \frac{h}{2} = 0; \tag{1}$$

being N, T and M the axial force, shear force and bending moment, respectively, τ and q denote the interfacial shear and peeling tractions, respectively, h represent the thickness of the beam and prime denotes differentiation with respect to coordinate x.

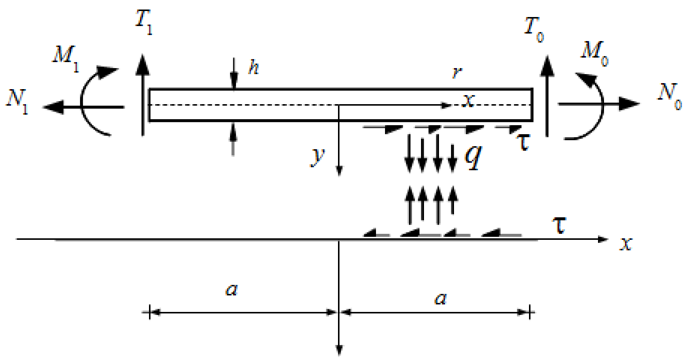

Figure 1. Free-body diagram of a beam bonded to a half-plane subjected to edge loads.

The constitutive laws (the slope of the beam cross section φ is positive if counterclockwise) give the following relationships:

$$\frac{M}{E_b I} = \varphi\,'; \qquad u\,' = \frac{N}{E_b A} + \varphi\,' \;\frac{h}{2}\;; \qquad v\,' = -\,\varphi + \frac{T}{G_b A_{eq}}\;; \tag{2}$$

where E_b denote the Young modulus of the beam, A and I are the area and the moment of inertia of the beam cross section respectively, G_b represents the shear modulo of the beam and A_{eq} is the shear area of the beam. The equilibrium conditions read:

$$N(x) = N_0 + \int_x^a \tau(s)ds\;; \qquad T(x) = T_0 + \int_x^a q(s)ds\;; \tag{3}$$

$$M(x) = M_0 + T_0\,(a-x) + \frac{h}{2}\int_x^a \tau(s)ds + \int_x^a q(s)(x-s)ds\;.$$

By replacing eqs (3) in expressions (2), the horizontal and vertical components $u_b'(x)$, $v_b'(x)$ of the beam strain field are found. The normal strains of the surface (i.e. for y = 0) an anisotropic half plane under 2D strain read (Johnson, 1985) for $|x| \le a$ are known from the Green fundamental solution:

$$u_s\,'(x) = -\frac{2(1-\nu_s^{\,2})}{E_s\,\pi}\int_{-a}^{a}\frac{\tau(\xi)}{\xi - x}\,d\xi + \frac{(1+\nu_s)(1-2\nu_s)}{E_s}\,q(x)\,;$$

$$v_s\,'(x) = -\frac{2(1-\nu_s^{\,2})}{E_s\,\pi}\int_{-a}^{a}\frac{q(\xi)}{\xi - x}\,d\xi - \frac{(1+\nu_s)(1-2\nu_s)}{E_s}\,\tau(x)\,. \tag{4}$$

The interfacial stresses within the contact region $|x| \le a$ are suitably expressed as follows:

$$\tau(x) = E_s\,(a+x)^s\,(a-x)^s\sum_n C_n\,P_n^{(s,s)}(x/a)\,, \qquad \sigma_{yy}(x) = E_s\,(a+x)^s\,(a-x)^s\sum_n D_n\,P_n^{(s,s)}(x/a)\,. \tag{5}$$

The problem is solved by imposing the compatibility condition $u_b'(x) = u_s\,'(x)$ and $v_b'(x) = v_s\,'(x)$ among the strain components of the beam and those of the half-plane in N+1 collocation points $x_k = \cos[\pi k/(2(N+1))]$, beng k = 1, 2, ….. , N + 1. Thus, the system of 2 integral equations leads to an algebraic system of 2N+1 equations in the N+1 unknowns coefficients C_i (i = 1, 3, 5, …., 2N +1), and N coefficients D_j (j = 2, 4, …., 2 N) of the interfacial stresses (5). Once that the stresses (5) are known, the internal forces (3) and the displacement field of the system at the interface can be found.

References

[1] Johnson, K. L., "Contact Mechnics", *Cambridge University Press* (1985).
[2] Shield, T.W., Kim, K.S., "Beam theory models for thin film segments cohesively bonded to an elastic half space ", *Int. J. Solids Struct.*, **29(9)** 1085-1103 (1992).

Adhesive elastic periodic contacts: the role of interfacial friction and slab thickness.

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Keywords: Contact Mechanics, adhesion, friction.

The optimization of a large variety of rubber-made engineering components (e.g. tires, dampers, rubber seals) often is strongly influenced by phenomena taking place at the interface of rough surfaces, as sliding friction and adhesion. Usually the presence of friction force in adhesive sliding contacts leads to a reduction of the contact area, as the presence of tangential forces at the interface increases the amount of elastic (repulsive) energy stored at the interface. However very recent experimental investigation [1], have reported a completely different behavior, in that sliding friction may unexpectedly causes an increase of the contact area. This result clearly contradicts preexisting investigations carried out by different authors in the case of non-interacting contacts [2].

In order to shed light on this phenomenon, we employ a simplified model where both mutual elastic interactions as well as friction and adhesion effects can be easily addressed.

In particular, in this work we focused on the problem of an elastic slab in sliding contact with a rigid wavy substrate in presence of adhesion and friction, see Fig. 1, where an elastic layer of thickness h is interposed between a flat rigid plate (upper part) and a sinusoidal rigid substrate with wavelength λ (bottom part).

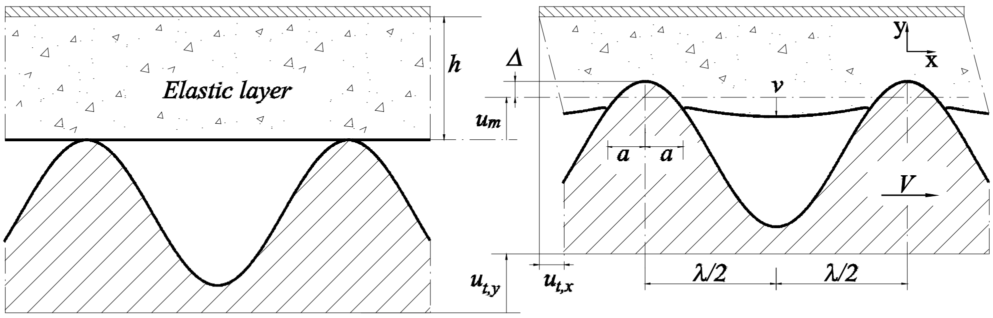

Figure 1: adhesive elastic sliding contact.

The whole problem has been formalized by generalizing the Green's function approach shown in [3] in the case of coexistence of normal and tangential loads. The elastic solution has then been obtained by numerically solving the coupled Fredholm equations of the first kind which govern the normal and tangential elastic fields.

The presence of adhesive interactions between bodies causes parts of the contact area to experience negative contact pressure (i.e. tensile stress). In those zones we assumed no frictional interactions, following the model proposed by [4, 5]. This, in addition to the finite thickness of the slab, leads to a non-linear elastic interaction between normal and tangential problem which has required an iterative solution procedure for the elastic problem.

The equilibrium solution is finally found for any given penetration. We found that conditions may occur under which, because of friction and mutual interaction between asperities, the contact area can increase depending on the relative importance of friction and adhesion.

These results are of utmost importance to explain the behavior of many different engineering components where friction and roughness play a dominant role, one example, among others, is the case of seals.

References

[1] Krick B.A., Vail J.R., Persson B.N.J., Sawyer W.G., “Optical In Situ Micro Tribometer for Analysis of Real Contact Area for Contact Mechanics, Adhesion, and Sliding Experiments”, *Tribol Lett*, **45**, 185–194 (2012).

[2] Savkoor, A.R., Briggs, G.A.D., “Effect of tangential force on contact of elastic solids in adhesion”. *Proc. R. Soc. Lond. Ser. A*, **356**, 103-114 (1977).

[3] Carbone G., Mangialardi L., “Analysis of the adhesive contact of confined layers by using a Green’s function approach”, *JMPS*, **56**, 684–706 (2008).

[4] Derjaguin B., “Molekulartheorie der äußeren”, *Reibung. Zeit. Phys. A*, **88**, 661–675, (1934).

[5] Gao J., Luedtke W.D., Gourdon D., Ruths M., Israelachvili J.N., Landman U., “Frictional Forces and Amontons' Law: From the Molecular to the Macroscopic Scale”, *J. Phys. Chem. B* **108**, 3410-3425, (2004).

Wednesday, April 1, 2015

Third Session

14:00-16:00

Viscoelastic contact problems: challenges and recent advancements

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Keywords: contact mechanics, viscoelasticity, friction.

Contact mechanics between linear viscoelastic solids is a crucial research field that is being approached by means of analytical, numerical and experimental techniques (see, for example, [1]). The reasons of this huge interest are mainly due to the necessity of a fully optimized design for a countless number of engineering applications (e.g. tires, dampers, rubber seals), involving rubber and rubber-based composites. In all these cases, the in-depth knowledge of viscoelastic dissipative phenomena is required to understand how the material properties and the macro- and micro-geometry of the interacting surfaces influence the final efficiency and durability.
For this purpose, a powerful tool has been developed to predict the behavior in terms of contact and friction for viscoelastic solids in steady-state sliding or rolling motions . This Boundary Element Methodology, already presented in [2] and experimentally validated in [3] for the case of smooth contact, has been successfully employed also to study the contact between viscoelastic rough surfaces [4]. Unlike other numerical methods, the main peculiarity of this approach is the possibility of handling real viscoelastic materials with more than one relaxations time [2]. This allows to study real materials, which cannot be modeled with an ideal one-relaxation-time formulation. This issue is particularly important in the case of rough contacts, where the frequency spectrum of the viscoelastic excitation covers a range of 6 orders of magnitude or more.
However, the main limitation of such an approach is related to the assumption of steady-state motion. Indeed, although in many cases, after a relatively fast transient state, experimental evidences show that the stationary solution provides accurate results, there are some situations where the time dependence has to be included. Under these conditions, for a linear viscoelastic material, the problem can be formulated by means of the following equation, relating the displacement to the interfacial normal stress distribution:

$$u(\boldsymbol{x},t)=\int_{-\infty}^{t} d\tau \int d^2x' \, J_1(t-\tau) J_2(\boldsymbol{x}-\boldsymbol{x}')\dot{\sigma}(\boldsymbol{x}',\tau) \qquad (1)$$

where $J_1(t)=\frac{1}{E_0}-\sum_{k=0}^{+\infty} C_k \exp(-t/\tau_k)$ and $J_2(t)=\frac{1-\nu^2}{\pi|\boldsymbol{x}|}$ with C_k positive coefficients (referred to as creep coefficients) and τ_k the relaxation times.
However, solving directly Eq. (1) is not straightforward. Indeed, this would require to manage a fine discretization both of the space domain and of the time one, thus resulting to be impossible when roughness is considered. Different strategies have to be considered.
In particular, we can focus our attention on the case of a rigid punch in periodic contact over a viscoelastic halfspace. In detail, we consider a punch moving according to the sinusoidal relation $x(t)=x_0 \sin(\omega t)$. For this particular case, it is possible to show that, because of the intrinsic periodicity of the problem and, consequently, of the solution, the following formulation parametrically dependent on the time can be obtained:

$$u(\boldsymbol{x},t) = \int d^2x'\, G(\boldsymbol{x}-\boldsymbol{x}',t)\sigma(\boldsymbol{x}',\tau) \tag{2}$$

Where $G(\boldsymbol{x},t)$ is the viscoelastic periodic Green's function. It is interesting to observe that in Eq. (2) the time domain is not explicitly present and this enables us to solve the problem by using the same techniques employed for the elastic and the stationary viscoelastic cases.
By means of this approach, we have studied the contact of rigid sphere in reciprocating contact over a viscoelastic halfspace. Figure 1 shows the pressure distribution for the fixed time t= π.

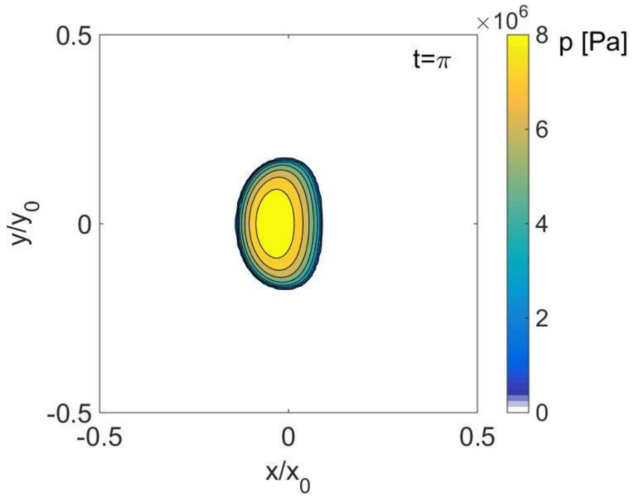

Figure 1: Contour plot of the pressure distribution for a sphere in rolling contact with a viscoelastic halfspace . The motion law is $x(t)=x_0 \sin(t)$ with $x_0=10^{-2}$ *m*; the time is t=π and the indentation is $\delta = 4\ 10^{-5}$ m.

In conclusion, although steady-state formulations can result really effective when dealing with viscoelastic contact problems, in some cases – e.g. in all the components where a reciprocating motion is present, like the vibration dampers- alternative procedures have to be considered to account for the role played by the time.

Acknowledgements

CP gratefully acknowledges also the support of Marie Curie IEF fellowship SOFT-MECH (grant number 622632).

References

[1] Persson B.N.J., Journal of Chemical Physics, 115, 3840 -3861, (2001).
[2] Carbone G., Putignano C., Journal of the Mechanics and Physics of Solids, 61 (8), pp. 1822-1834, (2013).
[3] Putignano C., Reddyhoff T., Carbone G., Dini D. Tribology Letters, 51 (1), pp. 105-113, (2013).
[4] Carbone G., Putignano C. , Physical Review E, 89, 032408, (2014).

Viscoelastic rolling/sliding contact problem with an heterogeneous material

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Keywords: Viscoelastic, Heterogeneity, Inclusion.

The frictionless rolling contact problem between a rigid sphere and a viscoelastic (VE) half-space containing one elastic inhomogeneity is solved. The problem is equivalent to the frictionless sliding of a spherical tip over a viscoelastic body. The inhomogeneity may be of spherical or ellipsoidal shape, the later being of any orientation relatively to the contact surface. The model presented here is three dimensional and based on semi-analytical methods. In order to take into account the viscoelastic aspect of the problem, contact equations are discretized in the spatial and temporal dimensions. The frictionless rolling of the sphere, assumed rigid here for sake of simplicity, is taken into account by translating the subsurface viscoelastic fields related to the contact problem. The Eshelby's formalism is applied at each step of the temporal discretization to account for the effect of the inhomogeneity on the contact pressure distribution, subsurface stresses, rolling friction and the resulting torque. A conjugate Gradient Method and the Fast Fourier Transforms are used to reduce the computation cost. Transient and steady-state solutions are obtained. Numerical results about the contact pressure distribution, the deformed surface geometry, the apparent friction coefficient as well as subsurface stresses will be presented, with or without heterogeneous inclusion.

An example of results is given in Fig. 1. The von Mises stress is plotted for a spherical inhomogeneity of dimensionless radius $r = 0.15a^*$ located at depth $dx_3^I = 0.3a^*$ within a viscoelastic half-space when a rigid sphere is rolling or sliding over the half-space. a^* is the initial contact radius for an homogeneous elastic half-space. Note that here the inhomogeneity is initially softer (relatively) than the viscoelastic matrix ($E^I = 0.4E^\infty$) and then becomes progressively harder than the matrix at very low strain rate ($E^I = 4E^0$). Results are presented at different time steps when the inhomogeneity is moving through the contact. The von Mises stress is plotted in the fixed coordinate system $R^0(O^0, X_1^0, X_2^0, X_3^0)$. At the initial time $t = 0$ the maximum value of the von Mises stress is maximum at the equator of the spherical inhomogeneity/matrix interface, reaching $0.8P_0$, significantly higher than the homogeneous solution ($0.6P_0$). It remains maximum in the vicinity of the inhomogeneity as far as this heterogeneity remains within the contact, however the absolute value drops quickly to $0.14P_0$ for $t = \tau/2$ while moving to the bottom side of the inhomogeneity/matrix interface. When the inhomogeneity moves away from the contact ($t = 2\tau$) the maximum of the von Mises stress corresponds to $0.1a^*$ located at $x_1/a^* = 1$ in the viscoelastic matrix. This observation is consistent since the peak of pressure for an homogeneous viscoelastic body is located at the same position.

References

[1] Koumi K.E, Chaise T., Nelias D., "Rolling contact of a rigid sphere / sliding of a spherical indenter upon a viscoelastic half-space containing an ellipoidal inhomogeneity", submitted.

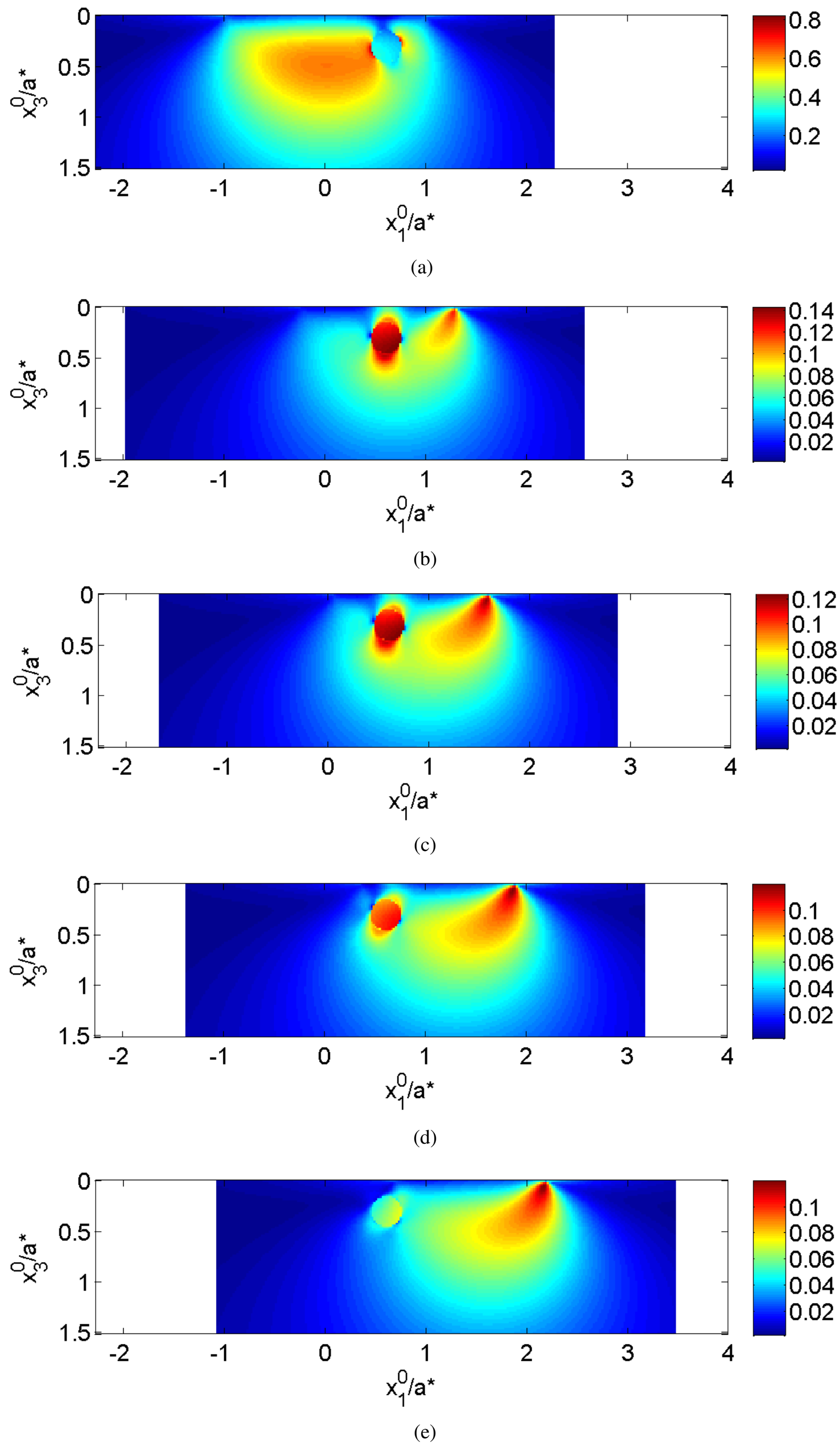

Figure 1: Von Mises stress field at different time steps when the viscoelastic half-space contains a spherical inhomogeneity of radius $r = 0.15a^*$ located at depth $dx_3^I = 0.3a^*$ ($\gamma = 0.4$); prescribed normal penetration $\delta = 0.1mm$ and dimensionless speed $v\tau/a_0 = 0.6$; (a) $t = 0$, (b) $t = \tau/2$, (c) $t = \tau$, (d) $t = 3\tau/2$, (e) $t = 2\tau$.

Numerical simulations of the frictionless contact between the rough surfaces of two elastic or viscoelastic bodies

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Keywords: Contact, Rough surfaces, Numerical simulations.

Numerical simulations of the contact between two rough surfaces are of great importance for understanding many mechanical and physical phenomena in solids. Such simulations are still a challenging task even in frictionless cases because the reel contact area is unknown in general and the accurate determination of this area requires very fine mesh [1]. When a great number of asperities are involved, the numerical simulation becomes excessively time consuming if the finite element method is used. This work presents the development of efficient approaches for simulating the contact between the frictionless rough surfaces of two elastic and viscoelastic solids by using a boundary element method. Periodically and randomly rough surfaces are considered.
In the case of periodically rough surfaces, the contact problem can be solved for a unit cell by using the periodical conditions [2]. With respect to finite element method, the number of elements is then drastically reduced and the calculation is accurate and fast. The method has been validated by comparing the numerical results with the available analytic solutions for one dimensional sinusoidal surfaces under both full and partial contact situations. For two dimensional wavy surfaces, the validation has been made firstly in the full contact situation where the analytic solutions are available. In the case of partial contact, comparisons have been made with some incomplete analytic solutions and other numerical solutions. Fig. 1 shows an example of the distribution of contact pressure.

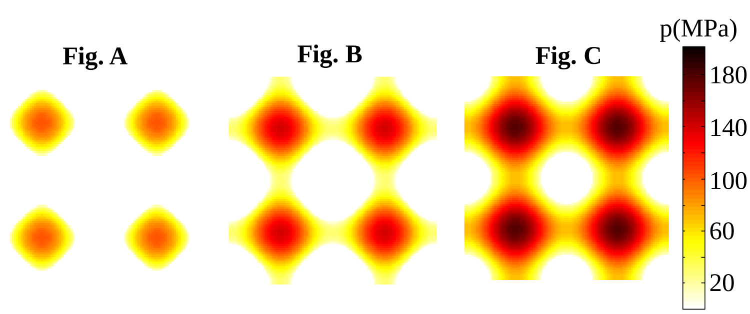
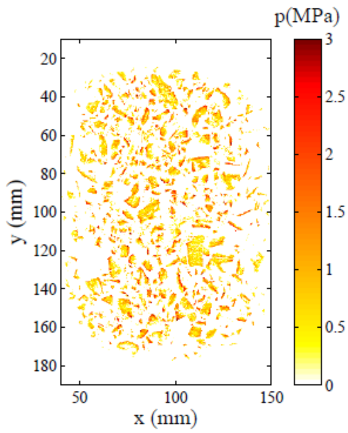


Figure 1: distribution of contact pressure between sinusoidal surfaces.



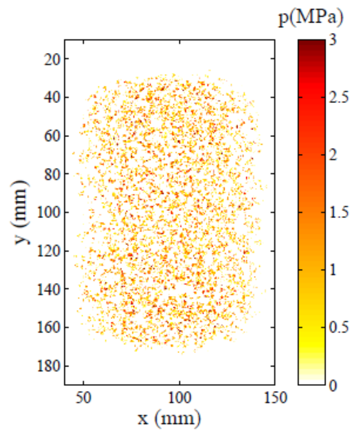


Figure 2: distribution of contact pressure between randomly rough surfaces.

For the contact between randomly rough surfaces, a multi asperity approach has been developed and successfully applied to the contact between a tire and a road surface [3, 4]. The method solves the contact problem in two steps. At the first step, the contact force at the summit of each asperity is calculated. The distribution of the contact pressure is calculated at the second step and the accuracy is improved by using an iterative method. Fig. 2 shows an example of distribution of the contact pressure. The method can be extended to the viscoelastic situation if the whole contact history is known. At each instant, the contribution of the anterior force to the current situation is represented by a time integral and the viscoelastic contact problem is then transformed into an elastic-like one. The method can also be extended to situations when adhesion forces or fluids are present between rough surfaces.

Acknowledgements
The research leading to these results has received funding from the french research agency ANR via Labex MMCD.

References

[1] B.N.J. Persson. Contact mechanics for randomly rough surfaces. Surface Science Reports, 61, 201-227 (2006).

[2] K. Houanoh, H.P. Yin, Q.C. He. Numerical analysis of the contact between frictionless elastic wavy surfaces by using boundary element method. IWCMM24, 1-3 october 2014, Madrid, Spain.

[3] J. Cesbron, H.P. Yin, F. Anfosso-Lde, D. Duhamel, D. LeHoudec, Z.Q. Feng. Numerical and experimental study of multi-contact on an elastic half-space. International Journal of Mechanical Sciences, 51, 33-40 (2009).

[4] G. Dubois, J. Cesbron, H.P. Yin, F. Anfosso-Lde. Numerical evaluation of tyre/road contact pressures using a multi-asperity approach. International Journal of Mechanical Sciences, 54, 84-94 (2012).

Sliding contact of a spherical indenter and a viscoelastic base with molecular adhesion

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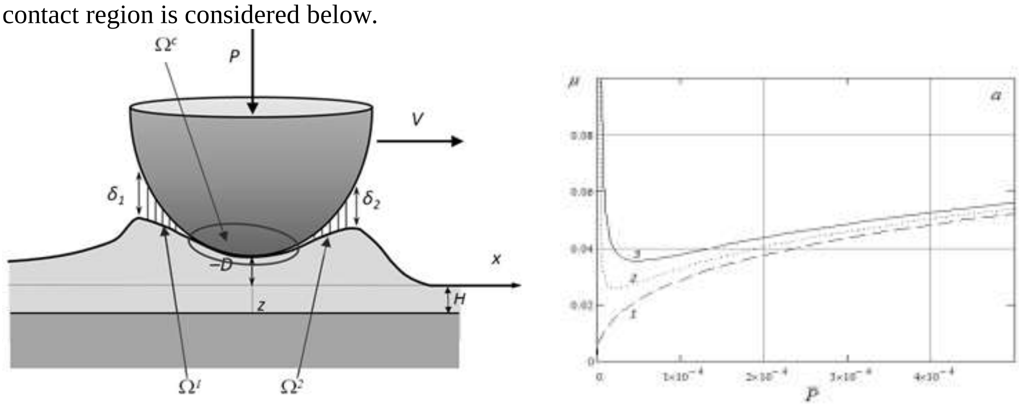
Keywords: contact mechanics, adhesion, viscoelasticity.

Imperfect elasticity of contacting bodies, their surface properties such as surface energy and surface roughness, play an important role in contact interaction and in relative sliding resistance. Normal contact of viscoelastic solids with adhesion, described by extended JKR theory, was studied in [1]. The combined effect of viscoelasticity and molecular adhesion in 2-D sliding contact of single and periodic systems of asperities and a viscoelastic base was analyzed in [2,3] under the assumption that characteristics of the surface adhesion are constants. However, it was experimentally shown in [4] that these characteristics change their values during the contact interaction. To study this effect, the 3-D contact problem for a spherical indenter sliding over the viscoelastic foundation with adhesion described by the various parameters in front and behind the contact region is considered below.



(a) (b)

Figure 1: Scheme of the contact (a) and the dependence of the friction coefficient μ on the dimensionless load $\bar{P}$ for various values of adhesive pressure: $p_1 = p_2 = 0$ (1), $p_1 = p_2 = 0.68 \cdot 10^{-4}\,Pa$ (2), $p_1 = 1.35 \cdot 10^{-4}\,Pa$, $p_2 = 0.68 \cdot 10^{-4}\,Pa$ (3)

A spherical rigid indenter slides over a viscoelastic layer of thickness H with a constant velocity V in the direction of the x-axis (Fig.1a). The mechanical properties of the viscoelastic layer are described by the linear 1-D model:

$$w + T_\varepsilon \frac{\partial w}{\partial t} = \frac{(1-\nu^2)H}{E}\left(p + T_\sigma \frac{\partial p}{\partial t}\right) \qquad (1)$$

Here p and w are the pressure and displacement on the boundary of the viscoelastic foundation, E is the Young modulus, ν is Poisson's ratio, T_ε and T_σ are the retardation and relaxation times, respectively.

To take into account the adhesive (molecular) attraction between the surfaces, introduce the negative adhesive stress $p = -p_a(\delta)$ acting on the boundary of the viscoelastic foundation (δ is the value of a gap between the surfaces). The Maugis-Dugdale model is used in which the dependence of the adhesive stress on the gap between the surfaces has a form of one-step function [5]:

$$p_a(\delta) = \begin{cases} p_0, & 0 < \delta \le \delta_0 \\ 0, & \delta > \delta_0 \end{cases} \quad (2)$$

Here δ_0 is the maximum value of the gap for which the adhesive attraction acts. The surface energy γ is specified by the relation:

$$\gamma = \int_0^{+\infty} p_a(\delta)\, d\delta = p_0\delta_0 \quad (3)$$

It is assumed that the $p_0 = -p_2$ in front of the contact (region Ω_2 in Fig.1a) if the gap does not exceed the value δ_2 and $p_0 = -p_1$ behind of the contact (region Ω_1 in Fig.1a) if the gap does not exceed the value δ_1.

The strip method is used to solve the problem. Analytical relations for the contact pressure and displacements of the viscoelastic foundation were obtained. A numerical algorithm was developed to determine the shape of the contact region and adhesive zones.

Based on the calculation results, the dependences of the contact characteristics and the friction coefficient on a sliding velocity, mechanical characteristics of the viscoelastic body, and the characteristics of the surface adhesion are analyzed. In particular, it follows from the study that increasing of the surface energy behind of the indenter due to contact interaction leads to the increase of the contact and adhesive regions, and to the increase of the friction force (Fig.1b).

The results are used to estimate the effect of adhesion in sliding contact of rough bodies, and to evaluate the influence of surface adhesion characteristics on mechanical component of friction force.

Acknowledgements

The research leading to these results has received funding from the Russian Science Foundation (grant 14-29-00198).

References

[1] Johnson K.L. "Contact mechanics and adhesion of viscoelastic spheres", *Am.Chem.Soc.Symp.Ser.741, 24-41 (1998).*

[2] Makhovskaya Yu.Yu., "Sliding of viscoelastic bodies in the presence of adhesion *", J. Appl. Math and Mech,, **69** 334-344 (2005).*

[3] Goryacheva I.G., Makhovskaya Yu.Yu., " Modeling of friction at different scales *", Mechanics of Solids.Izv.RAS,3 117-127 (2010).*

[4] Chaudhury M.K. and Owen M.J. "Adhesion hysteresis and friction." *Langmuir, 9(1), 29-31 (1993).*

[5] Maugis. D. "Adhesion of spheres: the JKR-DMT transition using a Dugdale model". J. Colloid Interface Sci., 150, 243-269 (1991).

Wednesday, April 1, 2015

Fourth Session

16:30-18:30

Boundary element solution of contact problems in the presence of electric fields

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Keywords: Contact problems, electro-elastic materials, piezoelectricity, boundary element method.

Computational contact analysis between multifield materials (e.i. piezoelectric materials) are increasingly being considered due to the growing use of these materials in many different high technological applications such as actuators, sensors, piezo MEMS or engineering control equipments among others. Engineers needs numerical tools to simulate the surface damage, micro-cracks or stress concentrations due to the contact mechanics in the presence of electric fields.

Different numerical finite element schemes have been proposed to simulate contact and indentation elasto-piezoelectricicity under contact conditions. Quasistatic solid-fundation frictionless contact problems for electro-elastic and electro-visco-elastic materials were studied in [1, 2], and in [3, 4], including frictional contact conditions. Dynamic piezoelectric contact were considered in [5, 6, 7]. More recently [8] studied the quasistatic solid-fundation frictionless contact problems for electro-elastic materials, based on the assumption that the foundation is electrically conductive.

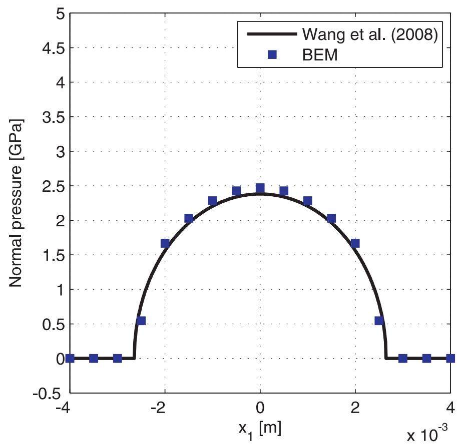
The Boundary Element Method (BEM) has never been used for modeling 3D frictional contact between fully anisotropic piezoelectric bodies in the presence of electric fields. The BEM is a very suitable numerical tool for interface interaction problems of multifield materials due to the number of degrees of freedom per node increases. The BEM considers only the boundary degrees of freedom involved in the problem and allows to obtain a very good accuracy with a low number of elements. Based in an explicit boundary element scheme presented in [9], the aim of this work is to provide a BEM formulation to simulate 3D frictional contact problems between piezoelectric solids. The boundary elements technique computes the electro-elastic influence coefficients, and the projection contact operators acting over the augmented Lagrangian guarantee the piezolectric contact restrictions fulfilment [10, 11]. The proposed formulation is applied to solve some benchmark examples (e.g. piezoelectric halfspace spherical indentation in Fig.1), and similar and dissimilar quasistatic contact problems between two piezoelectric solids.

Acknowledgements
The research leading to these results has received funding from the *Ministerio de Ciencia e Innovación*, Spain, and by the *Consejería de Innovación, Ciencia y Empresa*, Junta de Andalucía (Spain), through the research projects: DPI2013-43267-P and P12-TEP-2546, respectively.

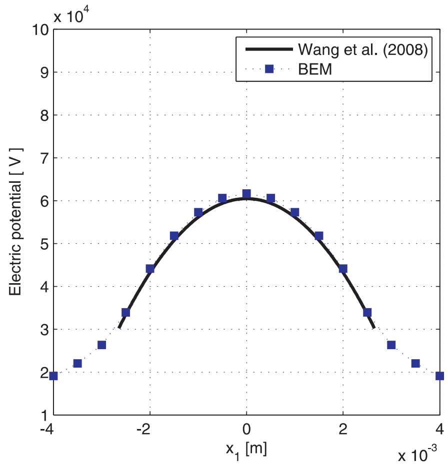
References

[1] Han, W., Sofonea, M., Kazmi, K., ”Analysis and numerical solution of a frictionless contact problem for electro-elastic-visco-plastic materials”, Comput. Methods Appl. Mech. Engrg., 196, 3915–3926 (2007).

[2] Barboteu, M., Fernndez, J.R., Ouafik, Y., ”Numerical analysis of two frictionless elastic-piezoelectric contact problems”, J. Math. Anal. Appl., 339, 905–917 (2008).



(a)



(b)

Figure 1: Comparisson between the boundary element solution and the analytical solution [12] for: (a) normal pressure distribution and (b) electric potential piezoelectric halfspace spherical indentation.

[3] Sofonea, M., Essoufi, E.H., ”Quasistatic frictional contact of a viscoelastic piezoelectric body”, Adv. Math. Sci. Appl., 14, 25–40 (2004).

[4] Sofonea, M., Kazmi, K., Barboteu, M., Han, W.M., ”Analysis and numerical solution of a piezoelectric frictional contact problem”, Appl. Math. Model., 36, 4483–4501 (2012).

[5] Barboteu, M., Fernndez, J.R., Tarraf, R., ”Numerical analysis of a dynamic piezoelectric contact problem arising in viscoelasticity”, Comput. Methods Appl. Mech. Engrg., 197, 3724–3732 (2008).

[6] Barboteu, M., Sofonea, M., ”Solvability of a dynamic contact problem between a piezoelectric body and a conductive foundation”, Appl. Math. Comput., 215, 2978–2991 (2009).

[7] Barboteu, M., Sofonea, M., ”Modeling and analysis of the unilateral contact of a piezoelectric body with a conductive support”, J. Math. Anal. Appl., 358, 110–124 (2009).

[8] Hüeber, S., Matei, A., Wohlmuth, B., ”A contact problem for electro-elastic materials”, ZAMM-Z. Angew. Math. Me., 93, 789–800 (2013)

[9] Buroni, F.C., Sáez, A., ”Three-dimensional Green’s function and its derivative for materials with general anisotropic magneto-electro-elastic coupling”, Proc. R. Soc. A, 466, 515 (2010).

[10] Rodríguez-Tembleque, L., Buroni, F.C., Abascal, R., Sáez, A.,” 3D frictional contact of anisotropic solids using BEM”, Eur. J. Mech. A. Solids, 30, 95–104 (2011).

[11] Rodríguez-Tembleque, L., Abascal, R. ”Fast FE-BEM algorithms for orthotropic frictional contact”, Int. J. Numer. Methods Eng., 94, 687–707 (2013).

[12] Wang, J.H., Chen, C.Q., Lu, T.J., ”Indentation responses of piezoelectric films”, J. Mech. Phys. Solids., 56, 3331–3351 (2008).

Study of roughness effect on elastic indentation of coated bodies

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Keywords: Elastic coatings, contact problem, roughness.

A two-level contact problem is proposed to study the effect of roughness of elastic coatings. The solution is based on using a function of additional displacements in a macro-level contact problem formulation. The dependence of additional displacements on nominal pressure is obtained from consideration of a periodic model of roughness. Analytical-numerical method, which is based oh Hankel integral transforms, boundary elements, and iteration procedure, is used to find macro-contact characteristics. The results are used for identification of elastic properties of relatively hard rough coatings from elastic indentation data.

Study of roughness is important for a contact of coated bodies especially for hard coatings with high resistance to plastic deformation. If the contact region is essentially larger than an average distance between asperities homogenization methods can be used to describe the roughness compliance for macro-contact problem. The modeling of roughness by one- and multi-level periodic system of asperities have been made by Goryacheva [1] to take into account the influence of asperities on macro characteristics of the contact with homogeneous half-space. Contact problem for two-layered elastic half-space and rough indenter is considered in [2]. In this study a model is presented to study the influence of a coating roughness on macro-contact.

A contact of an axisymmetrical smooth indenter and rough two-layered elastic half-space is under consideration (Fig.1). The surface roughness is described as a thin layer with thickness $h<<H$, where H is the coating thickness. Another condition is that $l<<a$, where l and a are the distance between asperities and a radius of the contact region. The boundary conditions are:

$$w(x,y) = C[p(x,y)] + A[p(x,y)], \qquad (1)$$

where $w(x,y)$ – normal displacements, $C[p(x,y)]$ – additional displacements caused by roughness, $A[p(x,y)]$ – displacements, which can be obtained from the contact problem solution for smooth two-layered half-space [3].

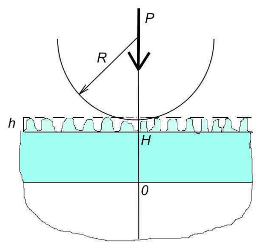
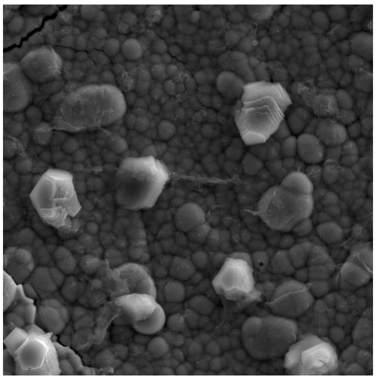
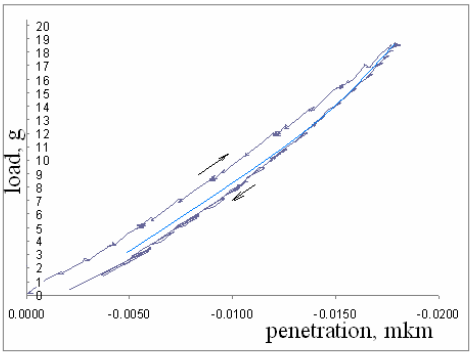


Figure 1: scheme of two-level contact.

For the case of $h<<H$ the additional displacements can be obtained from the periodic contact problem solution for homogeneous half-space as an analytical function [1]. The function should be presented as piecewise linear and used for the contact problem solution by boundary elements and iterations [2].

The influence of the asperities density on contact pressure and penetration is analyzed. It is obtained that for low density the influence is essential. Computation procedure is fast and can be used for calculation of large amount of contact problems, that's why it is used for identification of elastic properties of hard coatings with rough surfaces. Fig.2,a presents an example of roughness with similar highs of large asperities. The analysis of the surface geometry makes it possible to construct periodic system, which is used for the modeling. The Young modulus of the coating is 5% larger if the indentation data is treated taking into account roughness.





a b

Figure 2: roughness of a Ni-P coating (a); experimental and model curves for indentation of the Ni-P coating on steel substrate by steel ball (b)

Acknowledgements

The research leading to these results was financially supported by Russian Scientific Foundation (grant 14-19-01033).

References

[1] Goyacheva, I.G. *Contact Mechanics in Tribology*, Kluwer Academic Publishers, Dordrecht (1998).

[2] Torskaya, E.V., "Modeling of frictional interaction of a rough indenter and a twi-layered elastic half-space", *Physical Mesomechanics*, **15** (3-4), 245-250 (2012).

[3] Torskaya, E.V., Goryacheva, I.G., "The effect of interface imperfection and external loading on the axisymmetric contact with a coated solid'', *Wear*, **254**, 538-545 (2003).

The contact problem of a rigid flat punch indenting a couple-stress thermoelastic half-space

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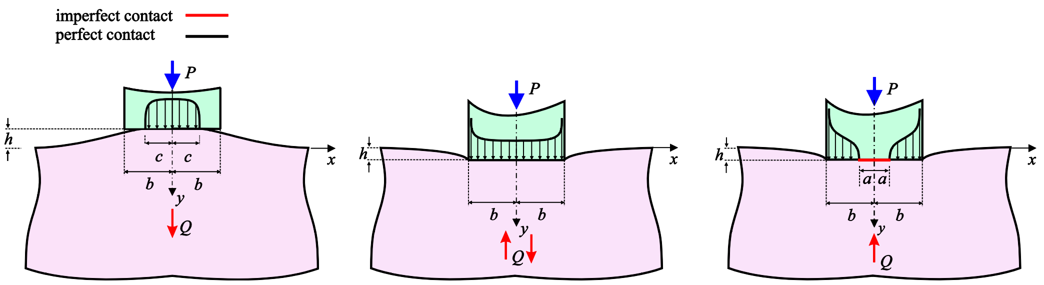
Keywords: Contact mechanics, thermal effects, micromechanics.

It is well known that thermo-elastic effects may have significant results upon the macroscopic response in the mechanics of contact. On the other hand, as the scales in the contact system reduce progressively (micro to nano–scales), the internal material lengths become important and their effect upon the macroscopic response cannot be ignored. The present work extends the classical contact solution for a rigid flat punch indenting a homogeneous elastic half–plane, where heat conduction is permitted [1], to the analogous case of an indented microstructured solid. The behavior of the indented material is modelled through the couple–stress theory elasticity, which introduces a characteristic material length, appropriately modified in order to incorporate the thermal effects [2]. The problem formulation is based on singular integral equations, resulted from a treatment of the mixed boundary value problems via integral transforms and generalized functions.



(a) *Separation* (b) *Perfect contact* (c) *Imperfect contact*

Figure 1: schematic representation of the plane problem. The rigid flat punch is pressed into the surface under the action of the force *P*. A temperature difference between the two bodies under contact induces a heat flux *Q* and the state of thermal stress is expected to alter the macroscopic contact characteristics, essentially modifying the contact area *b* from (b) *Perfect contact* to (a) *Separation* or (c) *Imperfect contact*. The macroscopic material response is governed by the micromechanical length introduced by the generalized continuum theory under consideration.

The solution shows that the thermo–mechanical response is strongly affected by the characteristic material length. Indeed, the conditions for which the type of contact passes from *perfect contact* to *separation* or to *imperfect contact* [2, 3] and the pressure distribution below the indenter change at varying the characteristic length-scale.

Acknowledgements

Thanasis Zisis and Francesco Dal Corso gratefully acknowledge support from the European Union FP7 project “HOTBRICKS - Mechanics of refractory materials at high–temperature for advanced industrial technologies” under contract number PIAPP–GA–2013–609758. Panos Gourgiotis gratefully acknowledges support from the European Union FP7 project “Modeling and optimal design of ceramic structures with defects and imperfect interfaces” under contract number PIAP–GA–2011–286110.

References

[1] Comninou, M., Barber, J. R., Dundurs, J. "Heat conduction through a flat punch". *Journal of Applied Mechanics*, **48** (4), 871-875 (1981)

[2] Nowacki, W., "Couple-Stress in the theory of thermoelasticity". Irreversible Aspects of Continuum Mechanics and Transfer of Physical Characteristics in Moving Fluids *IUTAM Symposia*, 259-278, (1968)

[3] Barber, J.R., "Contact problem involving a cooled punch". *Journal of Elasticity*, **8** (4), 409-423 (1978)

Numerical study of the frictionless contact problem between thermoelastic wavy surfaces

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Keywords: periodical roughness, thermoelastic surfaces, contact mechanics, boundary element method.

The contact between solids, in particular with rough surfaces, is a phenomenon met in many situations and applications. The thermoelastic behavior of the contacting bodies is responsible of different phenomena such as wear, crack and chemical reactions. Moreover this problem is very difficult to analyze, so efficient and accurate numerical methods are developed. The present work deals with the numerical analysis of the contact problem between a rigid wavy surface and a thermoelastic half-space, considering the frictionless condition and the periodical condition.

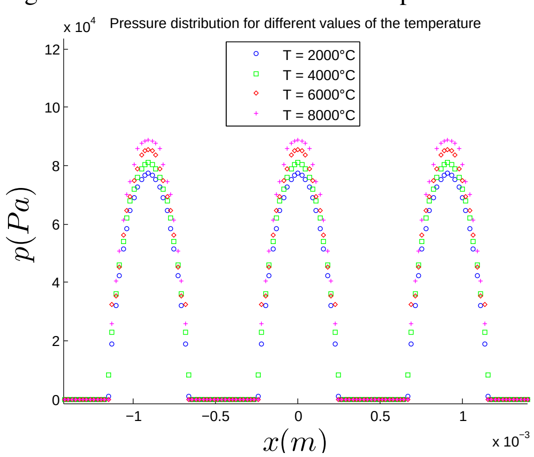

Figure 1 : pressure distribution on three asperities of a wavy surface containing $N = 121$ spherical asperities for different values of the temperature.

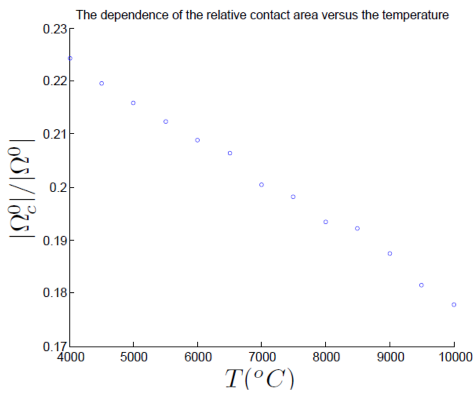

Figure 2 : the dependence of the relative contact area as a function of the temperature on a wavy surface containing $N = 121$ *spherical asperities.*

The boundary element method is very efficient for analysing problem because only the nominal contact area is meshed [1, 2, 3]. Besides, the periodical condition allows to reduce the number of unknowns implied in the numerical calculation. In order to validate the numerical method, the calculated contact area, pressure distribution and heat flow are compared to analytical solutions and results from numerical reference method [1, 5]. The numerical convergence of the difference between analytical solutions and numerical results is observed. The new method is efficient and very fast. Figure 1 and Figure 2 represent respectively the pressure distribution on a wavy thermoelastic surface containing $N = 121$ spherical asperities for four different values of the temperature and the evolution of the relative contact area with respect to the temperature. With the aim of highlighting the influence of roughness, periodical surfaces with spherical, sinusoidal, conical or pyramidal asperities are studied and results are compared to each other. This comparison shows that for periodical surfaces, the geometry of asperities and temperature variation has a great influence on contact areas, pressure distributions and heat flows. The present work can be extended to a large number of phenomena, such as frictional contact, problems with adhesion forces.

Acknowledgements
The research leading to these results has received funding from the Labex MMCD.

References

[1] Jang, Y.H., Cho, H., Barber, J.R., ”The thermoelastic Hertzian contact problem”, International Journal of Solids and Structures, 46, 4073-4078 (2009).

[2] Dubois, G., Cesbron, J., Yin, H.P., Anfosso-Lde, F., ”Numerical evaluation of tyre/road contact pressures using a multi-asperity approach”, International Journal of Mechanical Sciences, 54, 84-94 (2012).

[3] Cesbron, J., Yin, H.P., ”Contact analysis of road aggregate with friction using a direct numerical method”, Wear, 268, 686-692 (2010).

[4] K. Houanoh, H.P. Yin, Q.C. He. Numerical analysis of the contact between frictionless elastic wavy surfaces by using boundary element method. IWCMM24, 1-3 october 2014, Madrid, Spain.

[5] Barber, J.R., ”Indentation of the semi-infinite elastic solid by a hot sphere”, International Journal of Mechanical Sciences, 15, 813-819 (1973).

Thursday, April 2, 2015

First Session

08:30-10:30

Influence of plasticity on the real contact area during normal loading of rough surfaces

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Keywords: roughness, normal contact, plasticity.

The scientific community is currently giving a lot of attention to the evolution of the true contact area for two bodies under normal loading. In particular, Gaussian surfaces are particularly interesting due to their relative simplicity and generality, which allows a statistical characterization. Because the available theoretical models are approximate, numerical simulations have become an increasingly attractive alternative to provide a link between statistics of contact surfaces and mechanical properties. However, large surfaces with numerous asperities should be considered, which requires computationally efficient algorithms and the usage of computer clusters.

The use of the well known Jonhson's approximation allows to solve the problem using equivalent deformable-flat on rigid-rough systems using boundary-element approaches (BEM). The computational gain of discretizing only surfaces leads to efficient calculations and permits extensive surface refinement. Although the elasto-static normal contact has been widely explored, with many attempts to connect to available theories, the prediction of the true contact area for a given pressure is still debatable, especially at small loads.

At small pressures, the contacting area can be so small that the local concentrated stresses can yield. This inevitably leads to plastic deformations, which have to be modeled. Several works extended the boundary integral methods to account for elastic-perfectly plastic materials[1], where the maximal yield stress constraint was imposed on the surface pressure distribution.

We use a comparable strategy, to investigate the influence of such a yealding constraint over the contact problem during the normal loading employing a large set of representative rough surfaces[2]. The presented results focus on the true contact area and on the surface pressure distribution evolution. Furthermore, we will also provide the sub-surface stress evolution during the loading process, by using semi-analytical influence functions[3] derived with periodic boundary condition assumptions from the Fourier space.

Acknowledgements GA and JFM greatly acknowledge the financial support from the European Research Council (ERC- stg UFO-240332).

References

[1] A. Almqvist, F. Sahlin, R. Larsson, and S. Glavatskih. On the dry elasto-plastic contact of nominally flat surfaces. *Tribology International*, 40(4):574–579, April 2007.

[2] Vladislav A. Yastrebov, Guillaume Anciaux, and Jean-François Molinari. From infinitesimal to full contact between rough surfaces: evolution of the contact area. *arXiv:1401.3800 [cond-mat, physics:physics]*, January 2014.

[3] C. Jacq, D. Nélias, G. Lormand, and D. Girodin. Development of a three-dimensional semi-analytical elastic-plastic contact code. *Journal of Tribology*, 124(4):653–667, September 2002.

Optimization algorithms for the solution of the frictionless normal contact between rough surfaces

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Keywords: Frictionless normal contact, optimization algorithms, boundary element method, roughness.

This contribution revisits the fundamental equations for the solution of the frictionless unilateral normal contact problem between a rough rigid surface and a linear elastic half-plane using the boundary element method (BEM). After recasting the Linear Complementarity Problem (LCP) as a convex quadratic programming (QP) problem with nonnegative constraints, different optimization algorithms are compared for its solution: (i) the Greedy method with Conjugate Gradient (Greedy CG), (ii) the Alternating Direction Method of Multipliers (ADMM), and (iii) the Non-Negative Least Squares (NNLS) algorithm, possibly warm-started by accelerated gradient projection steps. The latter method is at least two orders of magnitude faster than the Greedy CG method. Moreover, due to the ability to include a warm start strategy, it is particularly useful when solving a sequence of contact problems with an increasing or a decreasing far-field displacement. Finally, we propose another type of warm start based on a refined criterion for the identification of the initial trial contact domain that can be used in conjunction with all the previous optimization algorithms. This method, called Cascade Multi-Resolution (CMR), takes advantage of physical considerations regarding the scaling of the contact predictions by changing the surface resolution. The method is very efficient and accurate when applied to real or numerically generated rough surfaces, provided that their power spectral density function is of power-law type, as in case of self-similar fractal surfaces.

Acknowledgements
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Influence of porosity content on homogenized mechanical properties

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Keywords: Porosity, Indentation, inhomogeneity, Semi Analytical Method.

The recent development of the semi analytical methods has led to numerous improvements in their capabilities. They allow now to perform fast and robust simulations of contact between semi-infinite bodies with either plastic behaviour [1] or when containing inhomogeneities [4]. The latter can be considered as inclusions without restriction about their nature or property. Thus, this has led to direct applications in, respectively, hetero-elasticity, poroelasticity [2], thermoelasticity [3] and elastic-plasticity when the inclusions get the nature of, respectively, material precipitations (carbides), voids (or defects), phase transformations of thermal origin and plastic strain (initial or not).

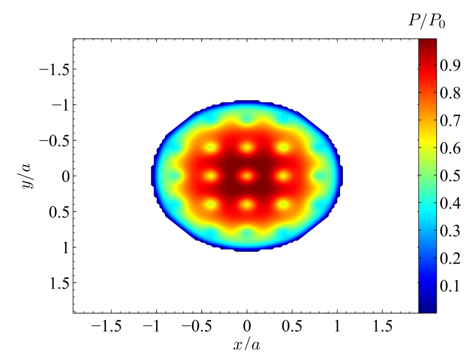
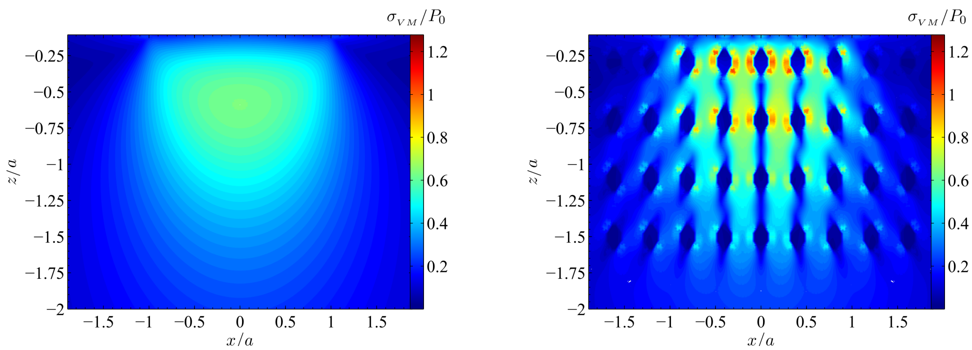


Figure 1: Contact pressure for a three dimensional distribution of porosities. P_0 is the Hertzian pressure and a the hertzian contact radius.

The present study investigates the effect of porosity volume fraction on the homogenized mechanical properties through performing an accurate indentation simulation on a porous body. The porosity content is driven by inclusion size and their distribution along the three space directions. Figure 1 shows the distribution of the contact pressure resulting from an indentation simulation on a porous body. Note that the contact area and pressure distribution depend on the matrix and inclusions material properties. In return, the material response is directly related to this pressure. Moreover, in situations when the porosities are subjected to this pressure, the local yield stress condition around them can be reached due to additional stress that they generated. Figure 2 shows the von Mises total stress for a porous body compared to an homogeneous one. The stress concentration is well visible as a function of porosities localization within the Hertzian stress field. Then plastic flow may occur under this local stress state. Based on equivalent indentation curves, the homogenized elastic and plastic properties can be determined thus showing the influence of the porosity content and distribution.



(a) (b)

Figure 2: Von Mises stress distribution resulting for indentation of spherical rigid tip on a homogeneous (a) or porous (b) elastic semi infinite body.

References

[1] Chaise, Thibaut and Nelias, Daniel and Sadeghi, F,. "On the effect of isotropic hardening on the coefficient of restitution for single or repeated impacts using a semi-analytical method", Tribology Transactions, pp. 714-722 (2011).

[2] Giraud, A., Huynh, Q., Hoxha, D., and Kondo, D.,"Application of results on eshelby tensor to the determination of effective poroelastic properties of anisotropic rocks-like composites", International Journal of Solids and Structures, pp. 3756 3772 (2007).

[3] Berryman, J. G.,"Generalization of eshelbys formula for a single ellipsoidal elastic inclusion to poroelasticity and thermoelasticity ". Physical review letters, 79(6), p. 1142.(1997).

[4] Leroux, J., Fulleringer, B., and Nelias, D., "Contact analysis in presence of spherical inhomogeneities within a half-space". International Journal of Solids and Structures, 47(22), pp. 30343049 (2010).

Micromechanical investigation of fracture of cold compacted powder

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Keywords: Powder Compaction, Discrete element method, Contact Mechanics.

Compaction of powders followed by sintering is a popular production route of components of hard materials, like cutting tools, and components of complex shape. First, a die, shaped as the final product, is filled with powder material. The powder is pressed and the resulting compact, the green body, is ejected. The green body has very weak mechanical properties but from sintering the component gets its full strength but it also gets changes in its dimensions dependent on the density distribution before sintering. It is also of importance that the green body is crack-free before sintering due to the fact that green body cracks introduces weak zones and crack initiation points in the final products. In this context, it is obvious that modeling of the compaction process is of great importance for the product development.

In this work, an experimental and numerical investigation of the fracture of powder compacts is presented with the aim of predicting cracks in the green body. The materials studied are two types of spry dried cemented carbide granules used in the industry. In the experimental part, powder compacts are pressed to different compact densities and later crushed in two different modes, axial and radial crushing. These modes are pictured in Figure 1 (a). A microscopy study of the fracture surfaces is performed which shows that both fracture of the individual powder granules and fracture of inter-particle contacts is important in order to describe the fracture process. An example of the fracture surface for the radially crushed compact is shown in Figure 1 (b).

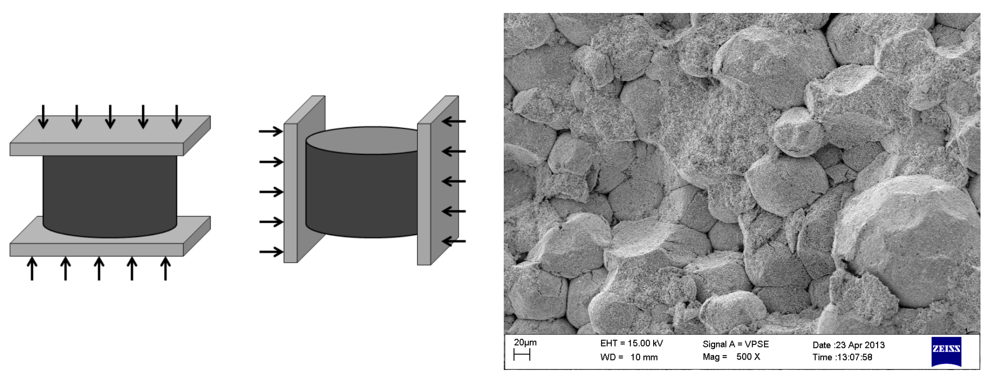

Figure 1: (a) Sketch of the crushing test setups. (b) Fracture surface of a radially crushed compact pressed to 45 % relative density

The whole experimental process, including compaction, unloading and compact crushing, is simulated using the Discrete Element Method (DEM) using an in-house developed code. The material behavior of the granules was determined in Olsson and Larsson [1] and it will be used together with the bonding models described in [2] to derive the relations between contact force and indentation depth needed in DEM. Crushing of the individual particles is introduced by formulating a particle fracture criterion using the forces acting on a particle. If the fracture criterion is fulfilled a fracture plane is introduced in the particle using all the forces acting on that particle. The post-fracture behavior is determined by assigning different stiffness reductions to each contact force depending on the angle between the contact force and the introduced fracture plane.

An example of the numerical results is shown in Figure 2 where axial crushing stress as function of the displacement of the crushing plate is studied and compared with experiments. As seen in the figure, the correct behavior of the crushing force is captured if fracture of the particles is included in the numerical model. A simulation without fracture of the particles, is also performed which shows a different behavior without the large drop in crushing stress at increasing displacement. Results for radial crushing and the shape of the fracture surfaces will also be presented.

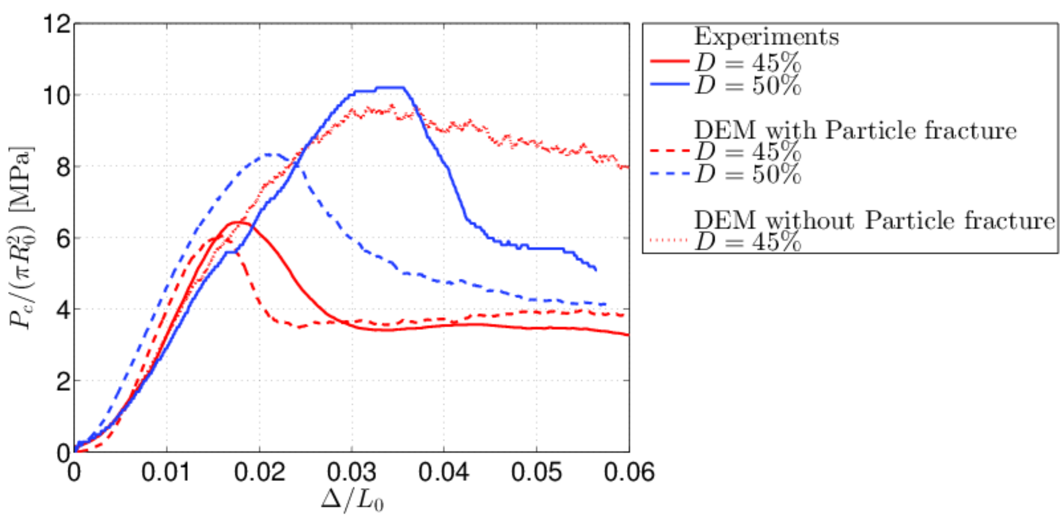

Figure 2: Comparison between the numerical simulations for the axial crushing load case.

Acknowledgments

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References

[1] E. Olsson, P.-L. Larsson, A Numerical Analysis of Cold Powder Compaction Based on Micromechanical Experiments, Powder Technol. 243 (2013) 71–78.

[2] E. Olsson, P.-L. Larsson, On Force-Displacement Relations at Contact Between Elastic-Plastic Adhesive Bodies, J. Mech. Phys. Solids 61 (5) (2013) 1185–1201.

Thursday, April 2, 2015

Second Session

11:00-13:00

A coupled impact problem for articular cartilage: Phenomenological modeling of damage in a biological tissue under dynamic loading

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Keywords: Blunt impact, articular cartilage, damage.

In recent years, much effort has been made on developing of impact testing methodologies for soft biomedical materials and biological tissues [1]. It was shown [2] that linear viscoelastic impact models are not capable of describing many sensitive features of observed impact phenomena for soft tissues [1, 3]. One of the most striking examples is the dramatical decreasing behavior of the coefficient of restitution as a function of the initial velocity of impactor documented in impact experiments on articular cartilage [3], whereas the linear viscoelastic impact models assert that it should be independent of the impact velocity. That is why, it was hypothesized [2, 3] that utilizing non-linear viscoelastic models in the impact material identification problem may shed light on the behavior of the output impact variables (in particular, coefficient of restitution) with variation of the input impact parameters (impactor mass and velocity).

In this paper, we concentrate on the other hypothesis that the damage accumulation during the impact of a biological tissue can be responsible of the observed peculiarities in the drop-weight experiments, which show increasing amounts of damage in the articular cartilage samples [4]. Following [5, 6], we employ a phenomenological theory of damage and assume that the damage results in the reduction of the elastic modulus. The damage evolution equation and the equation of motion of the impactor (with variable coefficients) constitute a coupled impact problem. This problem is analyzed for different damage evolution laws, and qualitative results for its solutions are formulated.

Acknowledgements
The research leading to these results has received funding from the European Union under the the FP7 IRSES Marie Curie grant TAMER No 610547.

References

[1] Kalcioglu, Z.I., Qu, M., Strawhecker, K.E., Shazly, T., Edelman, E., VanLandingham, M.R., Smith, J.F., Van Vliet, K.J., "Dynamic impact indentation of hydrated biological tissues and tissue surrogate gels", Phil. Mag. 91, 1339–1355 (2011).

[2] Argatov, I.I., "Mathematical modeling of linear viscoelastic impact: Application to drop impact testing of articular cartilage", Trib. Int. 63, 213–225 (2013).

[3] Edelsten, L., Jeffrey, J.E., Burgin, L.V., Aspden, R.M., "Viscoelastic deformation of articular cartilage during impact loading", Soft Matter 6, 5206–5212 (2010).

[4] Burgin, L.V., Aspde,n R.M., "Impact testing to determine the mechanical properties of articular cartilage in isolation and on bone", J. Mater. Sci. Mater. Med. 19, 703–711 (2008).

[5] Kachanov, L.M., "On the creep fracture time", Izv. akad. Nauk USSR 8, 26–31 (1958).

[6] Rabotnov, Y.N., Creep Problems in Structural Members, North-Holland, Amsterdam (1969).

Experimental validation of an asymptotic model to predict crack trajectories influenced by voids

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Keywords: Asymptotic elasticity, crack growth, fracture mechanics, X-FEM.

A two-dimensional asymptotic model (both plane stress and plane strain) for the determination of the crack trajectory interacting with elliptical inclusion has been developed in the analytical form by Movchan and his co-workers (see [1], [5], [7], [3] and [4]). Their model assumes a semi infinite crack growing quasi statically in an infinite, brittle, isotropic and linear elastic body under pure Mode-I loading (KII=0) and interacting with isolated defects 'far' from the straight trajectory that would be followed by the crack in the absence of disturbances.

Systematic experiments and computational simulations were performed to investigate the validity of the above asymptotic model to predict crack trajectories in brittle materials containing isolated voids. The experiments were performed by the quasi-static loading of v-shaped notched plates of brittle material under Mode-I. We have also identified both large holes and the dynamic regime where fracture surfaces show kinking and roughness as the limitations of the considered models.

Acknowledgements

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References

[1]. D. Bigoni, S.K. Serkov, M. Valentini and A.B. Movchan: "Asymptotic models of dilute composites with imperfectly bonded inclusions". *International Journal of Solids and Structures* **35** (24), 3239-3258 (1998).
[2]. A.B. Movchan. "Integral characteristics of elastic inclusions and cavities in the twodimensional theory of elasticity". *European Journal Applied Mathematics* **3**, 21-30 (1992).
[3]. A.B. Movchan and S.K. Serkov. "Elastic polarization matrices of polygonal domains". *Mechanics of Solids*, **26** (3), 63-68 (1991).

Multiscale characterization of complex surfaces: anti-reflective coatings, hydrophobic surfaces and fibrillar interfaces

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The description of roughness and of the morphology of natural and artificial surfaces has received a great attention by engineers and physicists due to its relevance in tribology, as well as for the investigation of adhesion forces and failure mechanisms taking place at interfaces [1-7].

In the present study we present novel experimental methods for the multiscale characterization of complex surfaces with a texture or with a microstructure, using the facilities available in the laboratory of the research unit MUSAM on Multi-scale Analysis of Materials at the IMT Institute for Advanced Studies Lucca.

As far as textured surfaces are concerned, we provide some insight into the possibility and limitations in applying random process theory and fractal modeling to describe the morphology of antireflective and hydrophobic rough surfaces. Usually the random process theory and fractal description are routinely applied to numerically generated surfaces. However, their applicability to real rough surfaces with texturing has not been scrutinized yet. Hence, we propose a morphological analysis of these real surfaces by using the confocal profilometer Leica DCM-3D to scan surfaces at different magnifications (see two images of a hydrophobic Ginkgo Biloba leaf in Fig. 1 at 10x and 100 x maginfications) and a Matlab software developed in house for their complete statistical and spectral characterization.

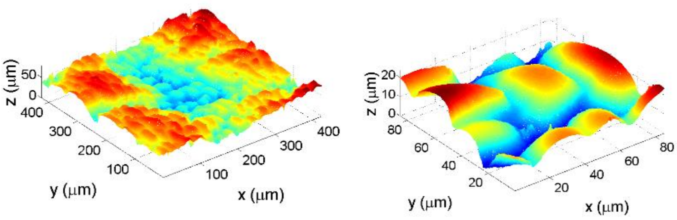

Figure 1: Ginkgo Biloba leaf scanned at 20x and 100x magnification.

We find that the distribution of asperity curvatures significantly deviates from the theoretically expected one, in spite of the fact that scale invariant properties of the PSD are almost preserved as in the numerically generated surfaces.

For fibrillar interfaces, on the other hand, we focus our attention on the experimental action of crack bridging forces exerted by cellulose fibrils in mechanically bonded paper tissues. In order to quantify adhesion of these fibrillar interfaces, a peeling test is performed by adapting the Deben MTEST 5000S tensile stage placed inside the scanning electronic microscope Zeiss EVO MA15. The proposed experimental setup allows to acquire pictures at the micro-scale while performing mechanical tests and relate the global response to the action of bridging fibers at the microscale (see Fig. 2). From the SEM image analysis we can also evaluate morphological parameters such as the radial distribution of fibers, essential for the characterization of the peeling process.

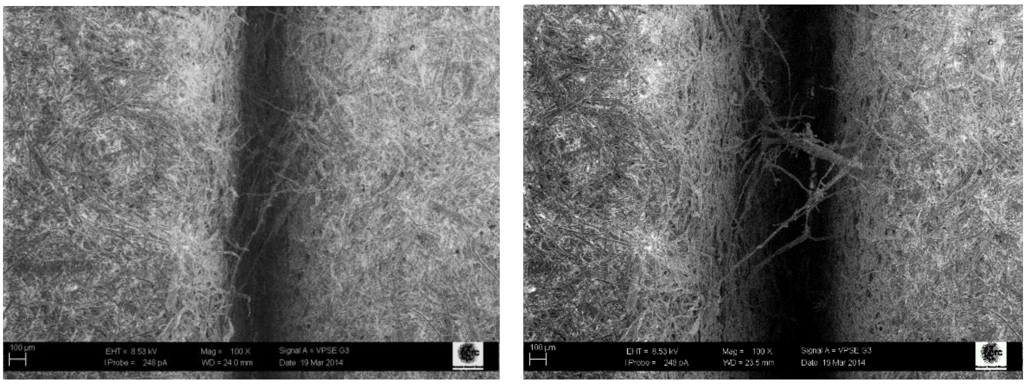

Figure 2: SEM images during peeling of paper sheets at 100x magnification.

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References

[1] Longuet-Higgins, M.S., “The statistical analysis of a random, moving surface”, *Philos. Trans. R. Soc.*, **A249**, 321, (1957).

[2] Nayak, P.R., “Random process model of rough surfaces”, *J. Lub. Tech.*, **93**, 398, (1971).

[3] Sayles, R.S, Thomas, T.R., “Surface topography as a nonstationary random process”, *Nature*, **271**, 431, (1978).

[4] Greenwood, J.A., “A unified theory of rough surfaces”, *Proc. R. Soc.*, **A393**, 133, (1984).

[5] Greenwood, J.A., Williamson, J.B.P., "The contact of nominally flat surfaces", *Proc. R. Soc. Lond. A*, **295**, 300-319 (1966).

[6] Greenwood, J.A., “A simplified elliptic model of rough surface contact” *Wear*, **261**, 191, (2006).

[7] Borri, C., Paggi, M., “Topological characterization of antireflective and hydrophobic rough surfaces: are random process theory and fractal modeling applicable?”, *Journal of Physics D: Applied Physics* **48**, 045301, (2015)”

Preliminary investigation of the cavitation damage in the conrod big end bearing of a high performance engine via a mass-conserving complementarity algorithm

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The conrod big end bearing is one of the most critical components of internal combustion engines; it is usually subjected to high dynamic loads and high sliding velocities between contacting surfaces are often involved. Therefore, the tribological behaviour of the conrod big end bearing is often one of the key elements to engine reliability, and it has been investigated both theoretically and experimentally in several contributions in the pertinent literature [1].

With the ever-increasing quest to improve engine performance, and the consequent increase of the rotational speed and combustion/inertial loads, the cavitation of the lubricant used in conrod big end bearings may play a crucial role in the assessment of bearing durability. To overcome problems related to film cavitation, palliatives such as the reduction of the clearance between the components, the increase of the oil supply pressure and/or the use of harder coating materials have been commonly adopted. However, such simple adjustments do not always represent a valid solution; further investigations are required in order to avoid cavitation damage occurrence.

The cavitation damage has been studied for more than a century both theoretically [2-5] and experimentally [6,7], and many attempts have been made to predict or measure the pressure spikes developing in the cavitated areas due to bubble collapse. Although a universally accepted theory seems not to exist, it is clear that the cavitation damage occurs due to the rapid and violent collapse of the vapour bubbles in the proximity of a solid boundary.

The aim of the present work is the preliminary evaluation of the damaging effect of the cavitation in a conrod big end bearing via elasto-hydrodinamic numerical analyses. In particular, the elasto-hydrodinamic problem is tackled using a formulation proposed by some of the authors [8,9] and based on the solution of the Reynolds equation suitably recast in terms of two complementarity variables, namely the hydrodynamic pressure, *p*, and the void-fraction, *r*:

$$\frac{\partial}{\partial x}\left[\frac{h^3}{6\mu}\frac{\partial p}{\partial x}\right] - 2\frac{\partial}{\partial t}[h] + 2\frac{\partial}{\partial t}[rh] - U\frac{\partial}{\partial x}[h] + U\frac{\partial}{\partial x}[rh] = 0$$

$$h = h_g + h_e = h_g + Cp \qquad (1)$$

$$p \geq 0$$
$$r \geq 0$$
$$p \cdot r = 0$$

The geometric thickness h_g considers the nominal profiles of the contacting surfaces while the compliance matrix *C*, employed to evaluate the elastic thickness h_e, is computed with a Finite Element model of the assembly.

The correct prediction of the cavitation and the location of the boundaries between cavitated and active areas is guaranteed by the complementarity nature of the two chosen variables and the approach naturally ensures mass conservation and unconditional convergence. The EHL problem is numerically solved using a weak formulation of the Reynolds equation based on the Finite Element Method.

The object of the study is the conrod big end bearing of a high performance motorbike engine. Comparative analyses involving different geometrical configurations of the crankpin/conrod assembly, and in particular of different shapes of the inner profile of the bearing, are presented.

A preliminary cavitation damage index is proposed, based on the variation in time of the void fraction at a certain location. The results obtained show a strong influence of the geometry of the bearing on the cavitation phenomenon. Experimental evidence qualitatively agrees with the numerical forecasts, thus corroborating the use of both the methodology and the cavitation damage index proposed in this contribution.

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References

[1] Booker J.F., Boedo S., Bonneau D., "Conformal elastohydodinamic lubrication analysis for engine bearing design: a brief review", *P. I. Mech. Eng. C-J. Mec.*, **224**, 2648–2653 (2010).

[2] Rayleigh L., "On the pressure developed in a liquid during the collapse of a spherical cavity", *Philos. Mag.*, **34**(200), 94–98 (1917).

[3] Plesset M.S., Chapman R.B., "Collapse of an initially spherical vapour cavity in the neighbourhood of a solid boundary", *J. Fluid Mech*., **47**(2), 283–290 (1971).

[4] Benjamin T.B., Ellis A., "The collapse of cavitation bubbles and the pressures thereby induced against solid boundaries", *P. Roy. Soc. Lond. A Mat.*, **260**(1110), 221–240 (1966).

[5] Kornfeld M., Survarov L., "On the destructive action of cavitation", *J. Appl. Phys.*, **15**, 495–506 (1944).

[6] Hobbs J.M., "Experience with a 20-kc cavitation erosion test. STP408: Erosion by cavitation or impingement", *ASTM, American Society for Testing Materials*, Philadelphia, 159–186, (1967).

[7] Lauterborn W., Bolle H., "Experimental investigations of cavitation-bubble collapse in the neighbourhood of a solid boundary", *J . Fluid Mech.*, **72**(2), 391–399 (1975).

[8] Giacopini M., Fowell M.T., Dini D., Strozzi A., "A Mass-Conserving Complementarity Formulation to Study Lubricant Films in the Presence of Cavitation", *Journal of Tribology*, **132**(4), 041702 (2010).

[9] Bertocchi, L., Dini, D., Giacopini, M., Fowell, M.T., Baldini, A., "Fluid film lubrication in the presence of cavitation: a mass-conserving two-dimensional formulation for compressible, piezoviscous and non-newtonian fluids", *Tribology International*, **67**, 61–71 (2013).